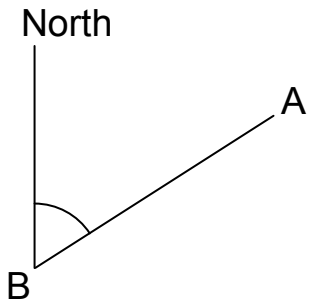
Trigonometry

Page	I can...	✓
1	... identify and label right-angled triangles	
3	... explain how to use the trigonometric ratios	
5	... calculate trigonometric ratios on a calculator	
7	... calculate a missing side when given a side and an angle	
10	... calculate a missing angle when given two sides	
14	... solve exam-style and worded problems	

Notes

Trigonometry and bearings

The bearing of A from B is the angle formed between the line AB and the straight line pointing towards North at B. The angle is measured clockwise and is given in three figures.



Three friends start sailing at a bearing of 230° for 30 miles.

How far due west are they from their original point?

What is the bearing from their current position back to where they started?

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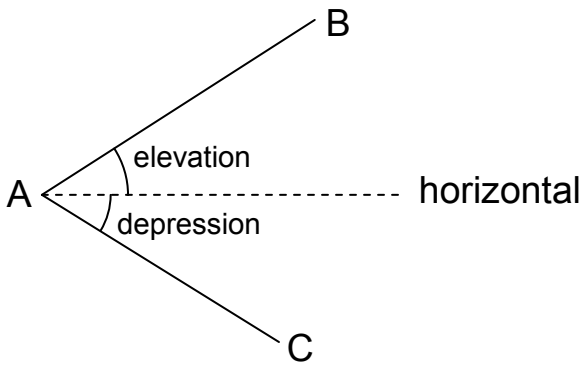
Angle of elevation

The **angle of elevation** of point B from point A is the angle formed between the line AB and the horizontal. Point A will be higher than point B.

The **angle of depression** of point C from point A is the angle formed between the line AC and the horizontal. Point C will be lower than point A.



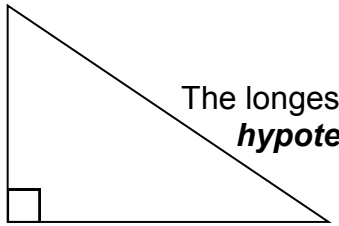
When a person looks up at the Eiffel tower from a distance of 99m, the angle of elevation to the top of the tower is 73°. When the same person looks up to the Big Wild Goose Pagoda from a distance of 30m, the angle of elevation is 65°.

Which is the tallest structure and by how much?

Right-angled triangles

A right-angled triangle is any triangle where one angle is 90°.

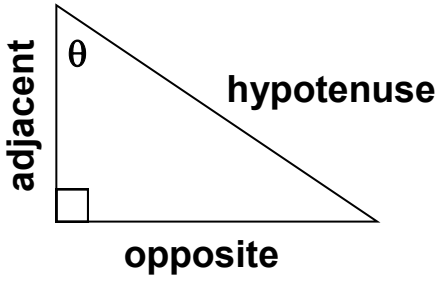
Since angles in a triangle sum to 180°, the other two angles must be acute (less than 90°). These two acute angles will be complementary (sum to 180°).

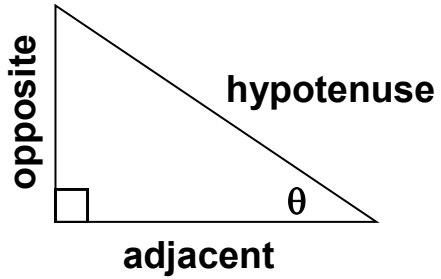


The longest side in a right-angled triangle is called the ***hypotenuse***, and is found opposite the right angle.

The two perpendicular sides only acquire specific names when they are in relation to a given angle (not the right angle). The side next to the angle is called the ***adjacent*** side; the remaining side is called the ***opposite*** side.

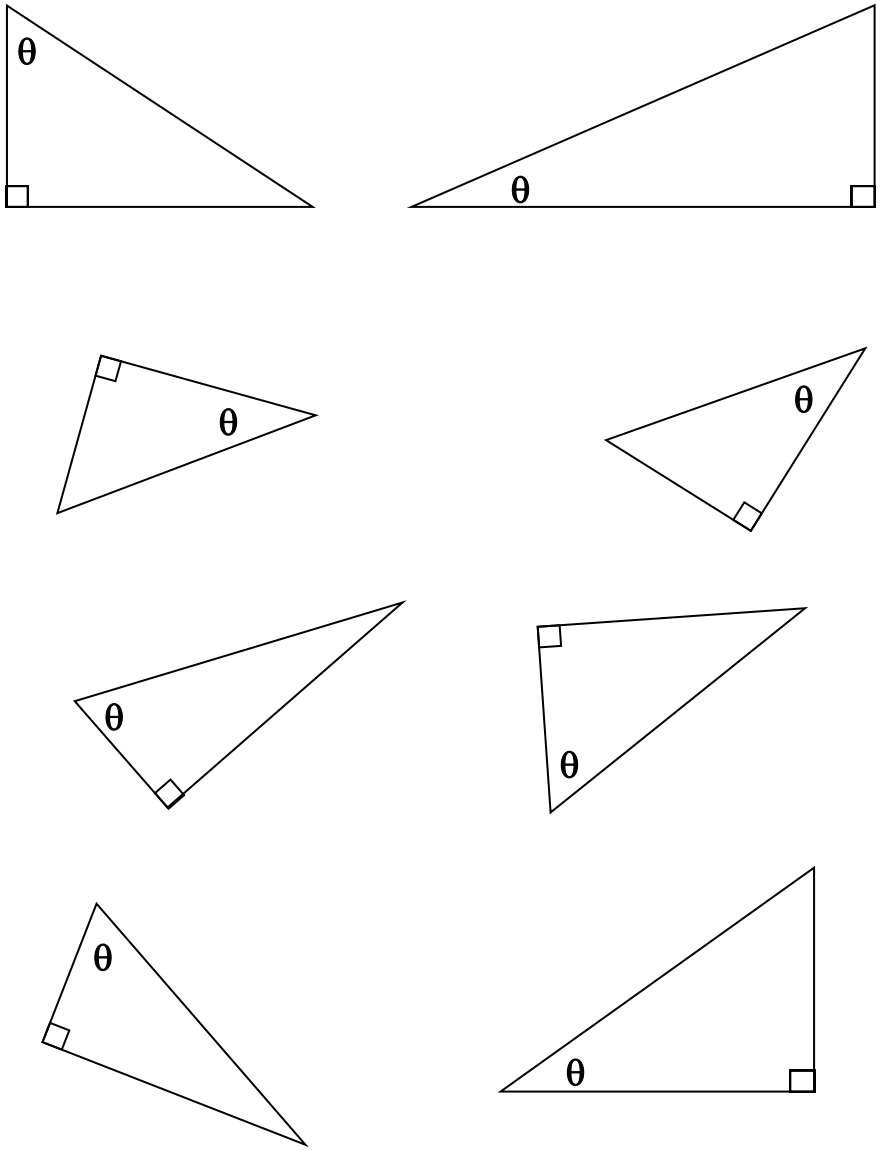
If we consider angle θ ...





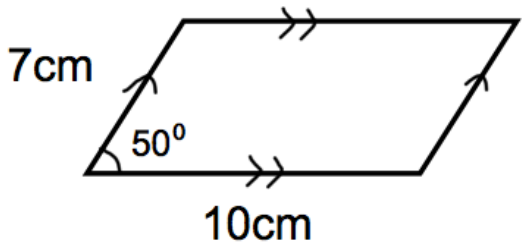
Labelling right-angled triangles

Label the **hypotenuse**, **adjacent** and **opposite** sides in these triangles. Remember that the adjacent and opposite sides depend on the position of the angle in question.



Exam style questions

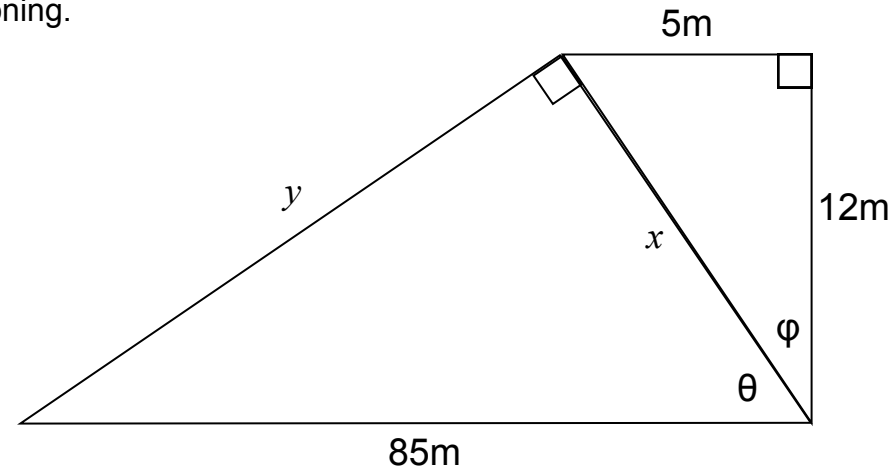
Calculate the area of the parallelogram.



Exam style questions

Find the missing sides x and y, and the missing angles θ and φ.

Label any other angles which are equal to θ or φ, explaining your reasoning.



Trigonometric ratios

Trigonometry links the sides of a triangle with its angles.

The word **trigonometry** comes from the Greek 'trigono' meaning 'triangles' and 'metro' meaning 'I measure'.

Trigonometry helps us calculate parts of any kind of triangle, but there are three main **trigonometric ratios**, which work with right-angled triangles:

sine	**cosine**	**tangent**
(sin)	**(cos)**	**(tan)**

Each ratio links an angle with particular sides:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

The **sine** of an angle is equal to the opposite side divided by the hypotenuse.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

The **cosine** of an angle is equal to the **adjacent** side divided by the **hypotenuse**.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

The **tangent** of an angle is equal to the **opposite** side divided by the **adjacent** side.

SOH-CAH-TOA

You need to memorise these ratios. '**SOH-CAH-TOA**' is a useful mnemonic acronym:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad s = \frac{o}{h}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad c = \frac{a}{h}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad t = \frac{o}{a}$$

Sine, cosine and tangents are **functions** (a process applied to an input to obtain an output). We can apply these functions using a scientific calculator.

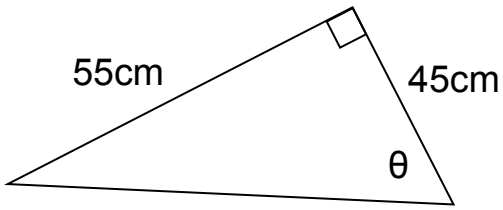
For example, using a calculator, find the sine of 30:

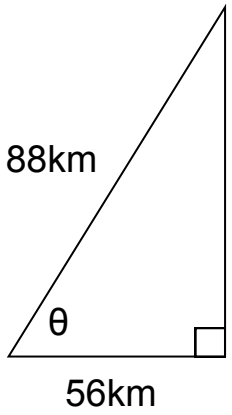
Type | sin | 3 | 0 | = |

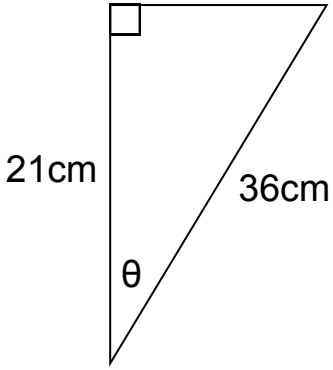
You should get a result of $^1/_2$.

Note: You will need to ensure your calculator is in **degree** mode. You should see 'D' or 'DEG' somewhere on the screen. Always check you are in the correct mode before starting work, especially before an exam!

Finding a missing angle





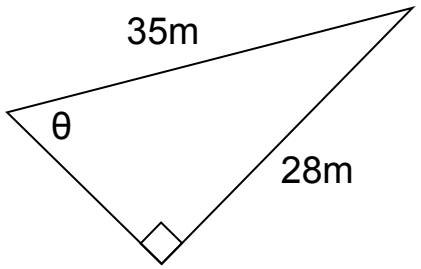


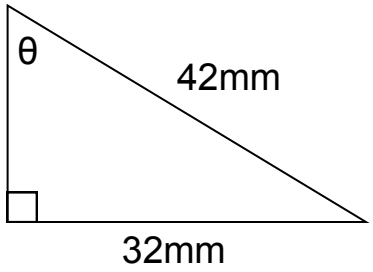
Finding a missing angle

Find the angle labelled θ in each of each of these triangles. Give your answers correct to three significant figures.

Remember to:

- identify which ratio to use
- substitute the values into the ratio
- apply to inverse function to find the angle.





Using a scientific calculator

Now try these:

sin(45) = ...	tan(23) = ...	cos(35) = ...
tan(88) = ...	cos(70) = ...	sin(10) = ...
cos (56) = ...	sin(28) = ...	tan(60) = ...
sin(90) = ...	cos(0) = ...	tan (45) = ...
sin (0) = ...	cos(90) = ...	tan(0) = ...

Q You should find that the sine and cosine values are never above one. Can you use SOH-CAH-TOA to explain why?

Both sine and cosine divide by the hypotenuse, which is the longest side. Therefore the result will always be a fraction smaller than one.

Q Try finding tan(90). What happens? Can you explain why?

If the angle in question was 90^{o}, the 'hypotenuse' would be perpendicular to the 'opposite' side, which is impossible in a triangle.

Substituting into SOH-CAH-TOA

Using the information given in the triangles below, substitute the values into the trigonometric ratios to find the sine, cosine and tangent of angle θ (correct to 2 decimal places).

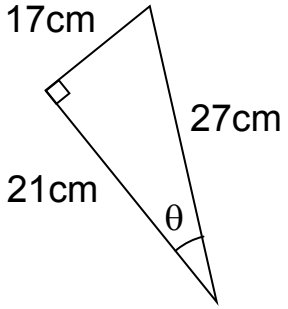
Remember to write down the ratio you are using first – this helps us to remember each ratio and check our work later.

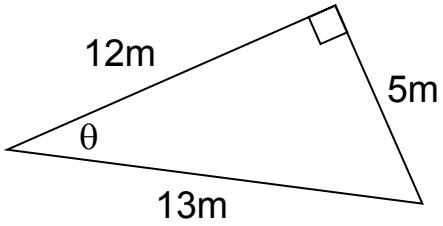
Example

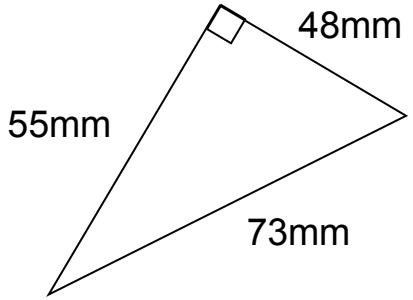
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{17}{27} = 0.63$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{21}{27} = 0.78$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{17}{21} = 0.81$$







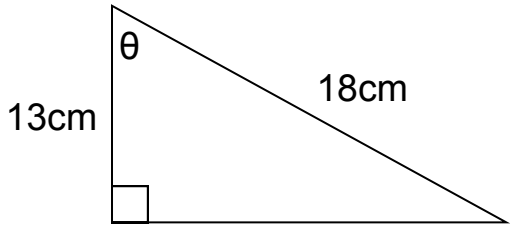
Finding a missing angle

You can now use the trigonometric ratios to find a missing angle when given two sides.

You can do this by:

- identifying which ratio to use
- substituting the values into the ratio
- applying the inverse of the function to each side.

Example



We have been given the hypotenuse and the side adjacent to the missing angle. Therefore we will use cosine:

$$\cos\theta = \frac{\text{adj}}{\text{opp}}$$

$$= \frac{13}{18}$$

$$\theta = \cos^{-1}\left(\frac{13}{18}\right)$$

$$= 46.2382...$$

$$= 46.2^{\circ}$$

Careful!

Make sure you take the inverse cosine of the whole of the right hand side. You could use brackets on your calculator to be sure.

Inverse functions

To calculate the size of an angle when given two sides of the triangle, we will need to 'separate' the angle from the sin, cos or tan function.

To do this, we must use the ***inverse*** of the sin, cos or tan functions.

Example

$$\sin\theta = 0.8$$

To get θ by itself, we must apply the inverse of sin (denoted by $\sin^{-1}$) to both sides of the equation:

$$\theta = \sin^{-1}(0.8)$$

To use the inverse sine function on our scientific calculators, we type:

shift	sin	0	.	8	=

$$\theta = 53.13010...$$
$$= 53.13^{\circ}$$

Find θ to three significant figures:

Sinθ = 0.2	cosθ = 0.66	tanθ = 1.35	tanθ = 7.3
θ = ...	θ = ...	θ = ...	θ = ...
Cosθ = 0.87	sinθ = 0.225	sinθ = 0.005	tan θ = 3.6
θ = ...	θ = ...	θ = ...	θ = ...
Cosθ = 0.112	cosθ = 0.5	sinθ = 0.0001	tanθ = 1
θ = ...	θ = ...	θ = ...	θ = ...

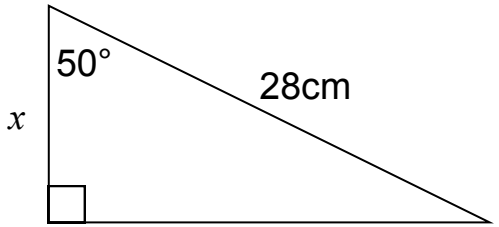
Finding a missing side

You can use the trigonometric ratios to find a missing side when given one side and one angle (other than the right angle!).

You can do this by:

- identifying which ratio to use
- substituting the values into the ratio
- rearranging to find the missing side.

Example



We have been given the hypotenuse, and asked to find the side adjacent to the given angle. Therefore we will use cosine:

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 50 = \frac{x}{28}$$
$$x = 28 \times \cos 50$$
$$= 17.998...$$
$$= 18.0\text{cm}$$

Careful!

'cos50' is one single term! It cannot be split by rearranging! Treat it as one object.

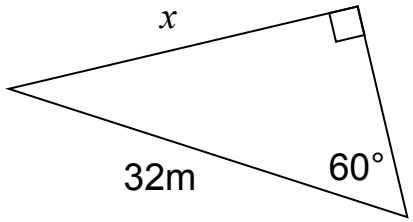
28 × cos50 is **not** the same as cos(50 × 28) – try it on your calculator!

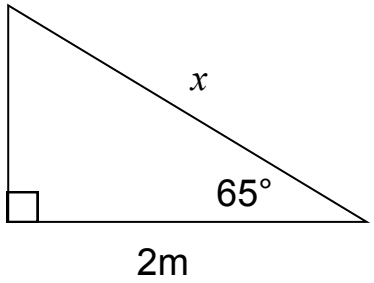
Finding a missing side

Find the side labelled x in each of each of these triangles. Give your answers correct to three significant figures.

Remember to:

- identify which ratio to use
- substitute the values into the ratio
- rearrange to find the missing side.





Finding a missing side