

Sketching quadratic functions

To do this, we first need to complete a table of values and then plot the points on the graph. Finally, join the plotted points with a **smooth curve**. The finished curve is called a **parabola**.

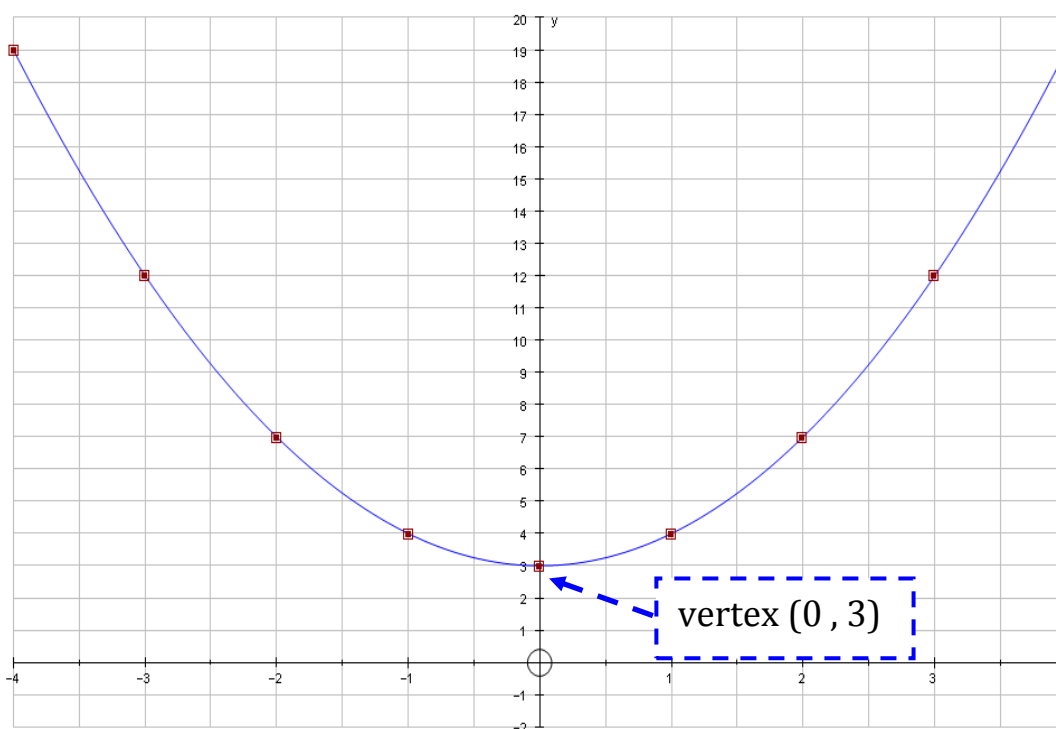
Worked example

Sketch $y = x^2 + 3$

$$\begin{aligned} x = -4 &\rightarrow y = (-4)^2 + 3 = 16 + 3 = 19 \\ x = -3 &\rightarrow y = (-3)^2 + 3 = 9 + 3 = 12 \\ x = -2 &\rightarrow y = (-2)^2 + 3 = 4 + 3 = 7 \\ x = -1 &\rightarrow y = (-1)^2 + 3 = 1 + 3 = 4 \text{ etc} \end{aligned}$$

x	-4	-3	-2	-1	0	1	2	3	4
y	19	12	7	4	3	4	7	12	19

It should look like this:



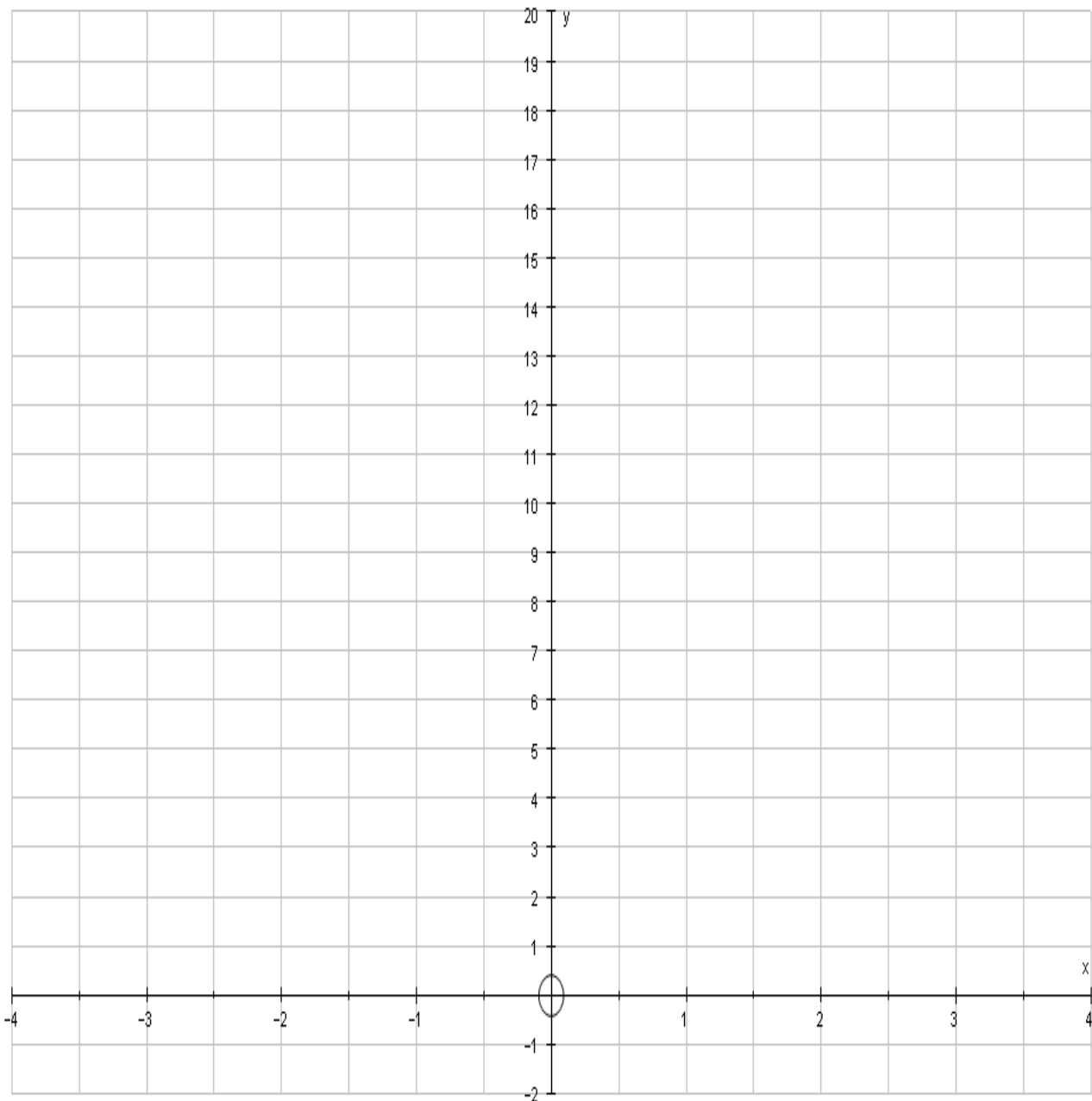
Notice the parabola is symmetrical, and the y -axis is the line of symmetry.

The **vertex** on this curve is at the lowest point and so it is called a **Minimum Turning Point**.

Exercise

1. Sketch $y = x^2$

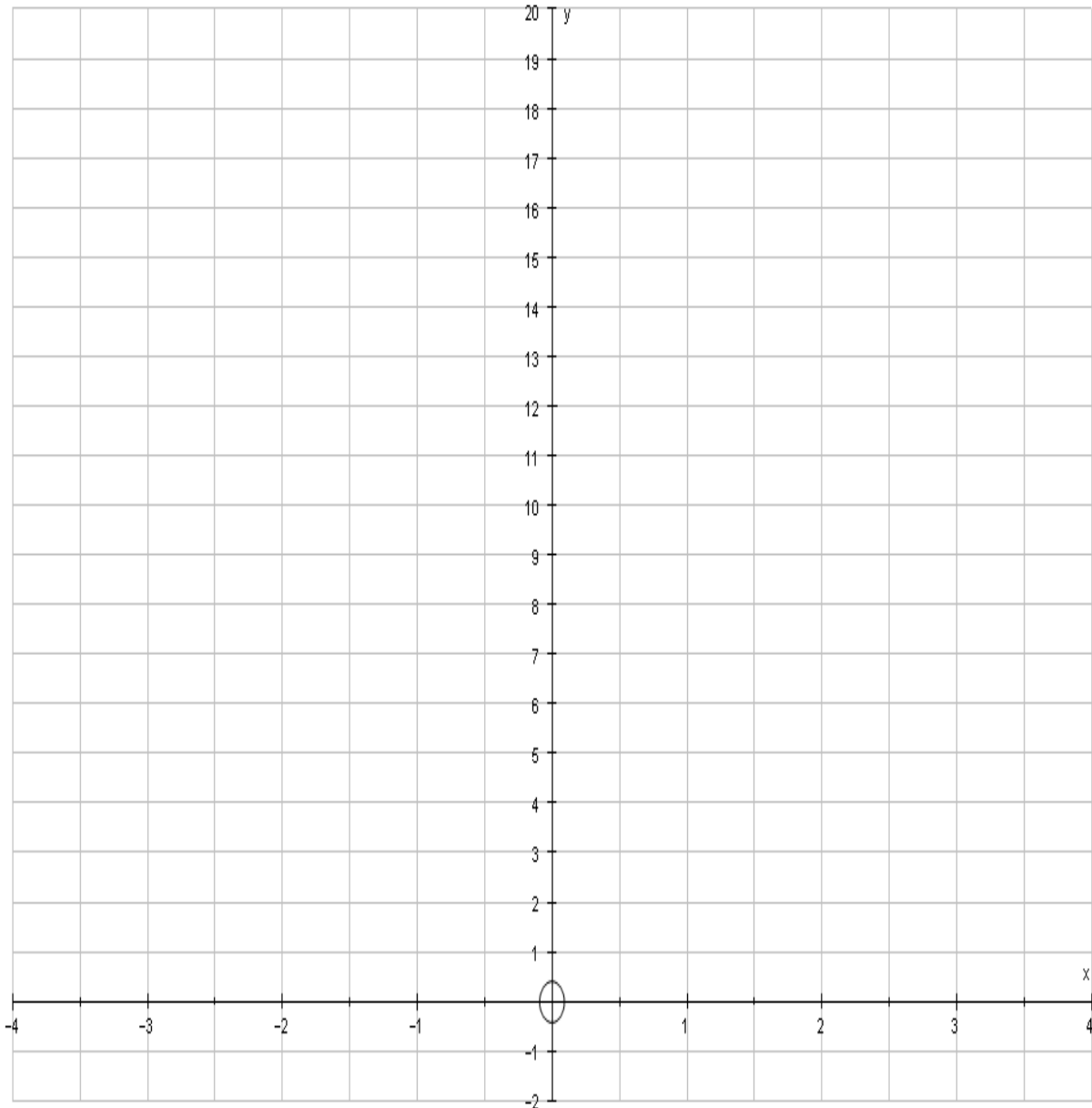
x	-4	-3	-2	-1	0	1	2	3	4
y									



The following questions are all based on $y = x^2$

2. Sketch $y = 2x^2$

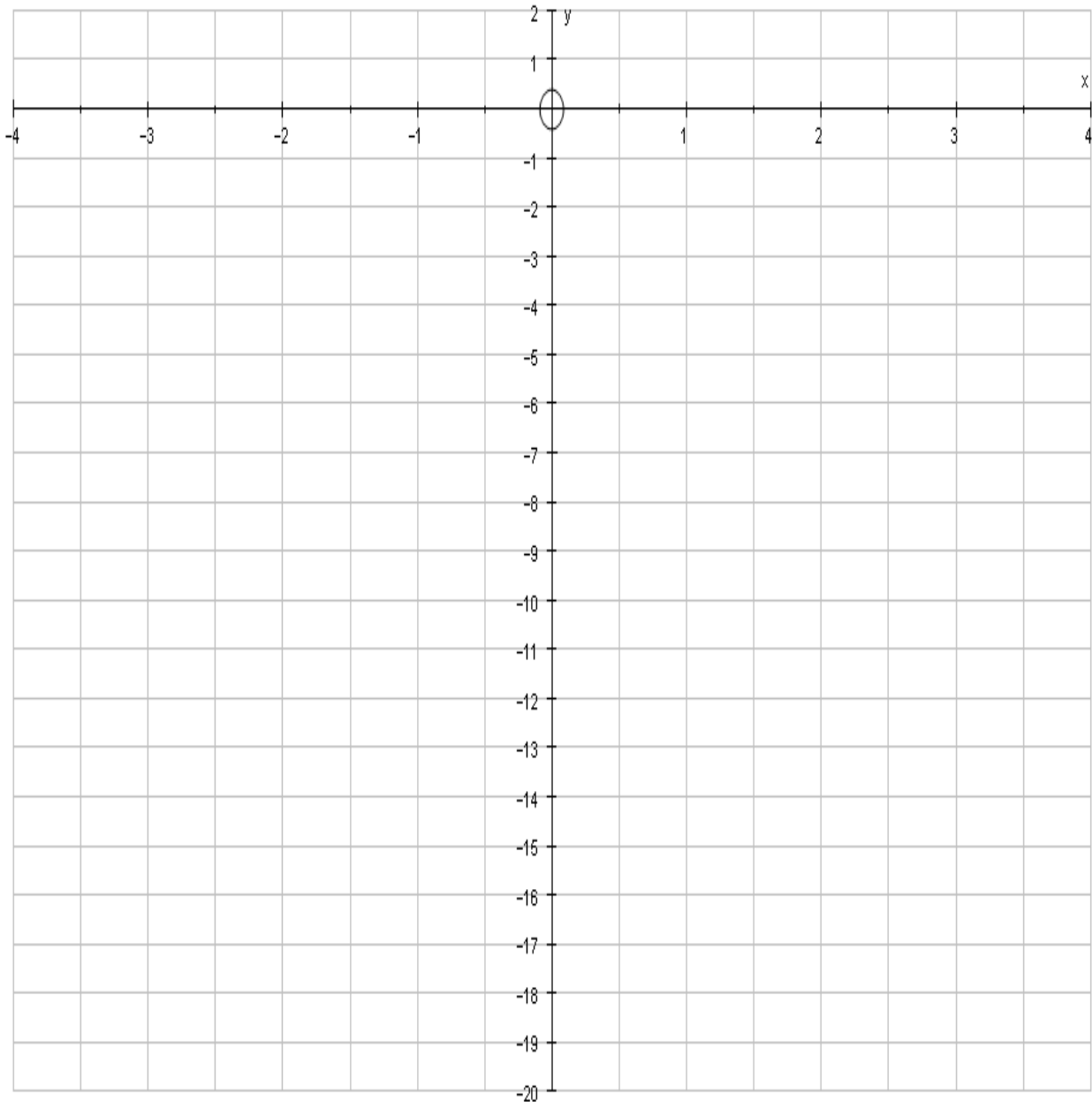
x	-3	-2	-1	0	1	2	3
y	18						



The curve $y = x^2$ has been stretched vertically by a scale factor of 2

3. Sketch $y = -x^2$ (This is not the same as $y = (-x)^2$)

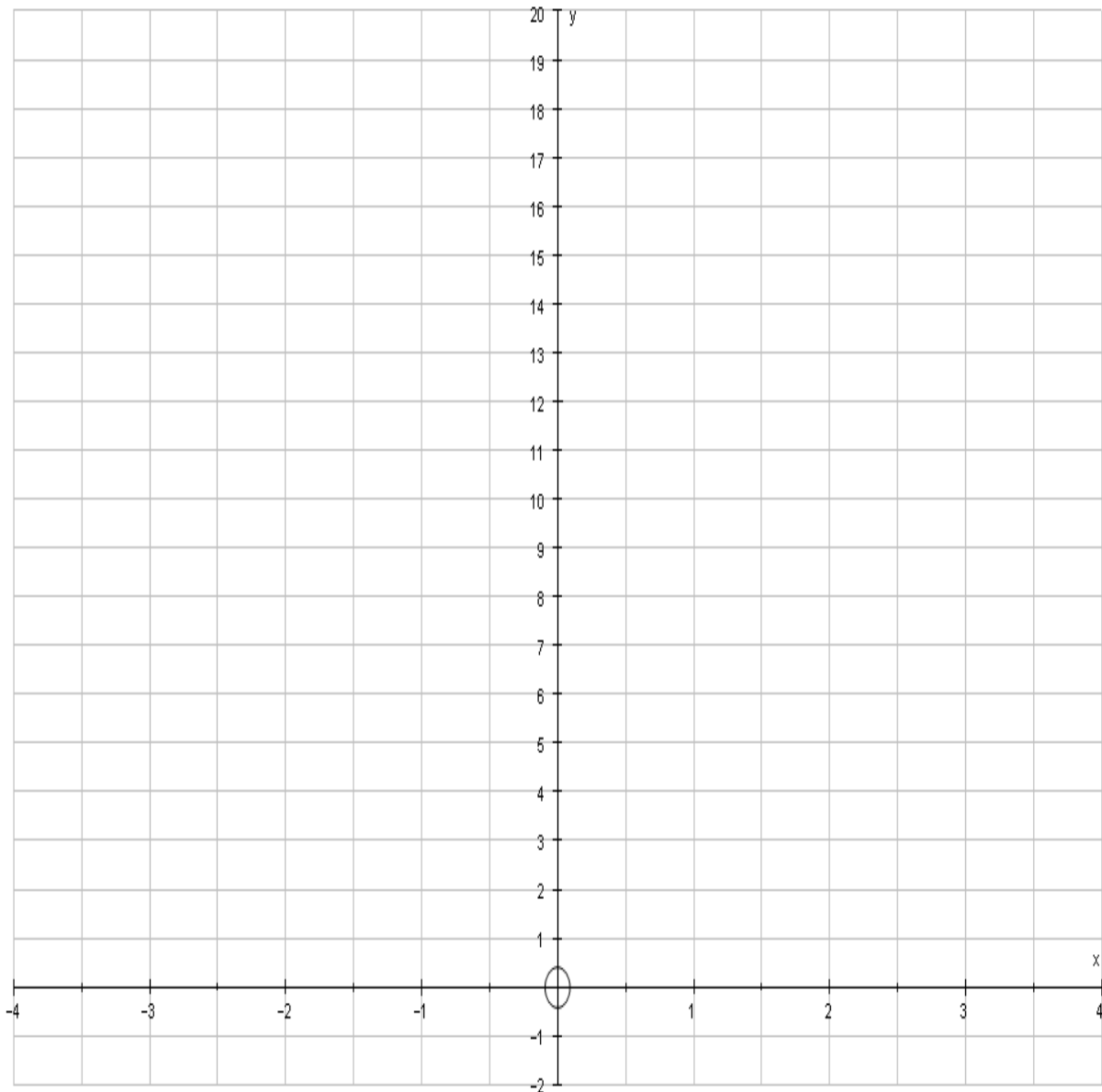
x	-4	-3	-2	-1	0	1	2	3	4
y								-9	



$y = x^2$ has been reflected in the x -axis. This has a **Maximum Turning Point**.

4. Sketch $y = x^2 + 2$

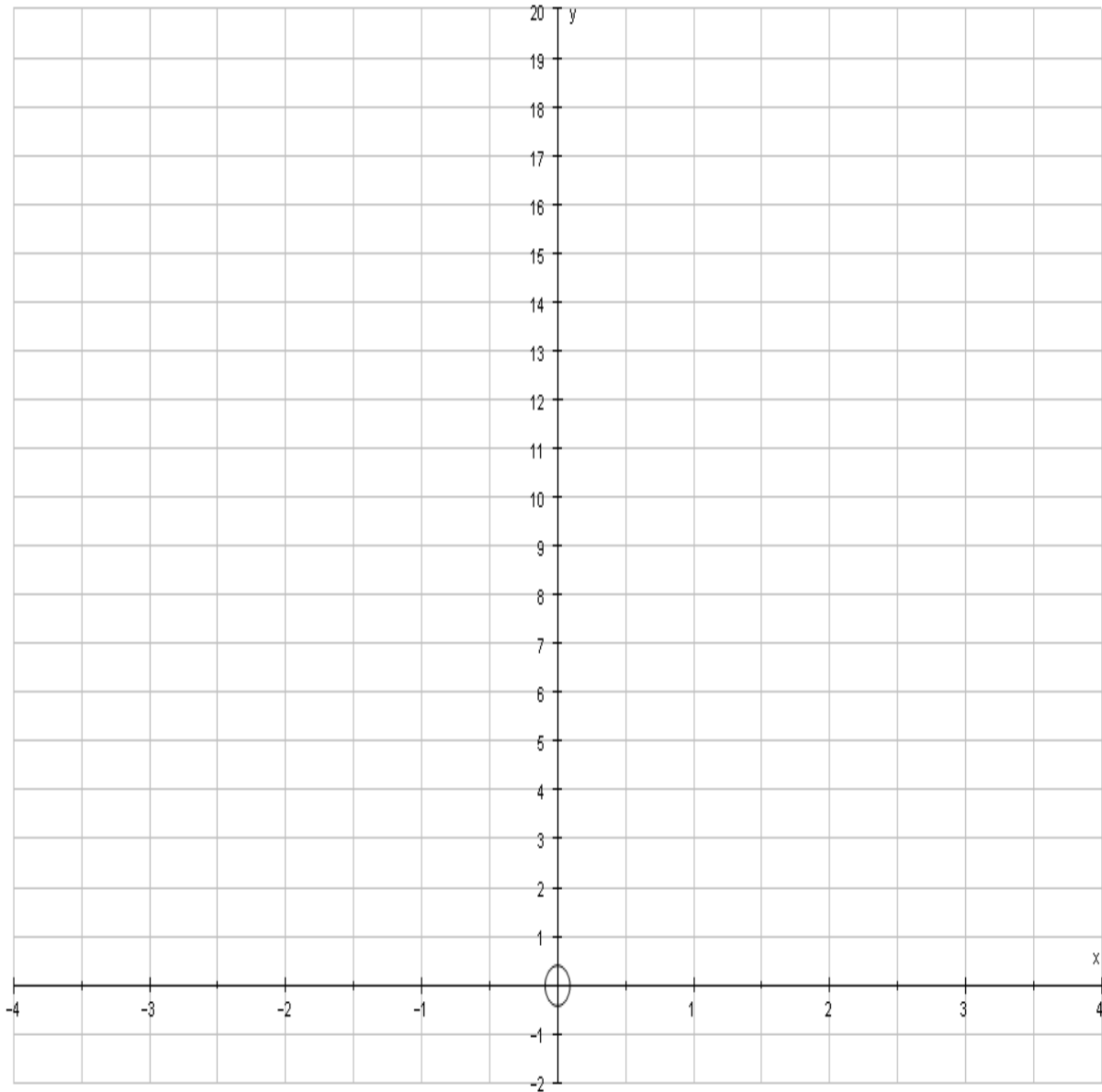
x	-4	-3	-2	-1	0	1	2	3	4
y									



$y = x^2$ has been translated vertically upwards by 2 units.

5. Sketch $y = x^2 - 1$

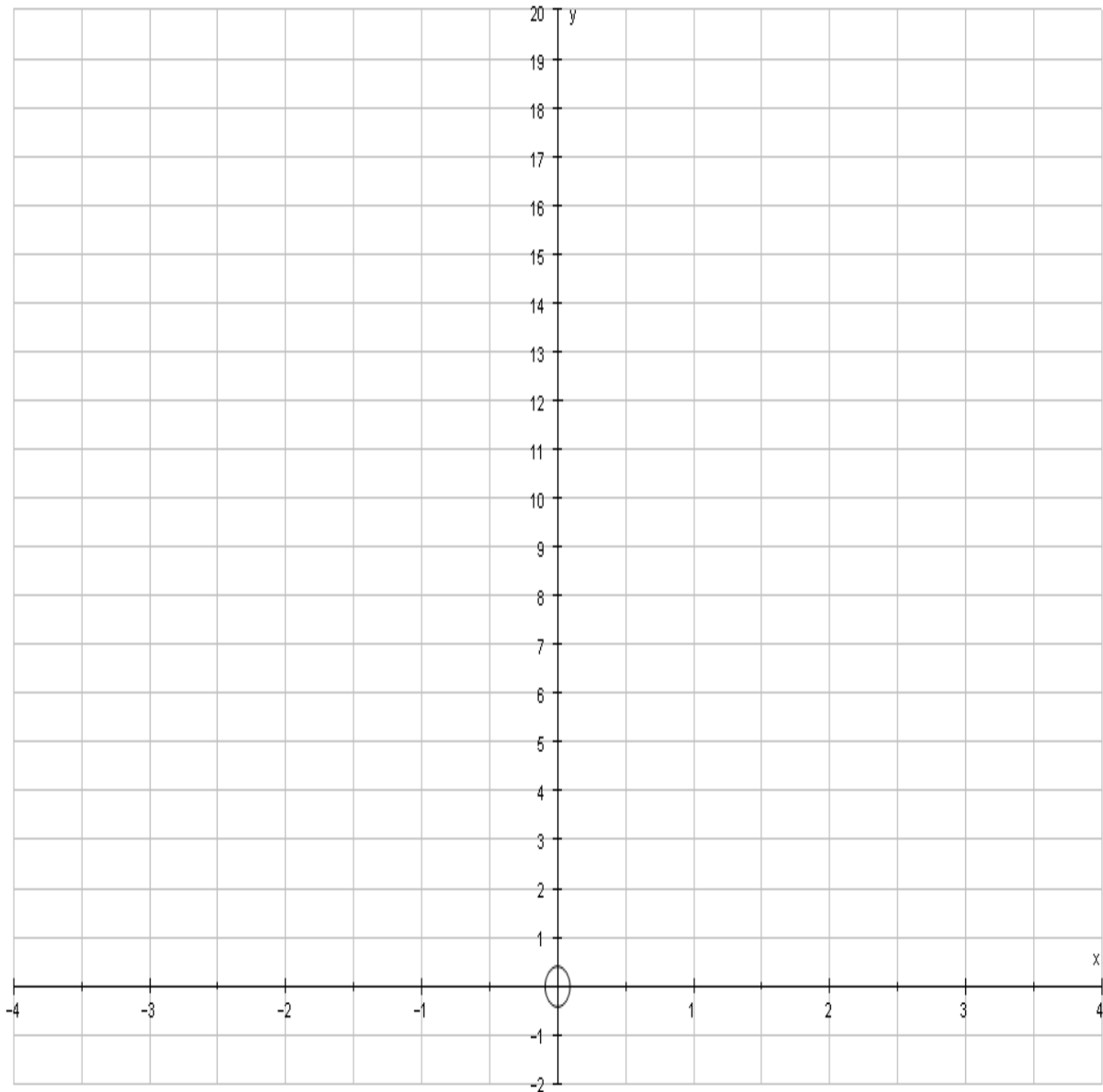
x	-4	-3	-2	-1	0	1	2	3	4
y						0			



$y = x^2$ has been translated vertically downwards by 1 unit.

6. Sketch $y = (x + 1)^2$

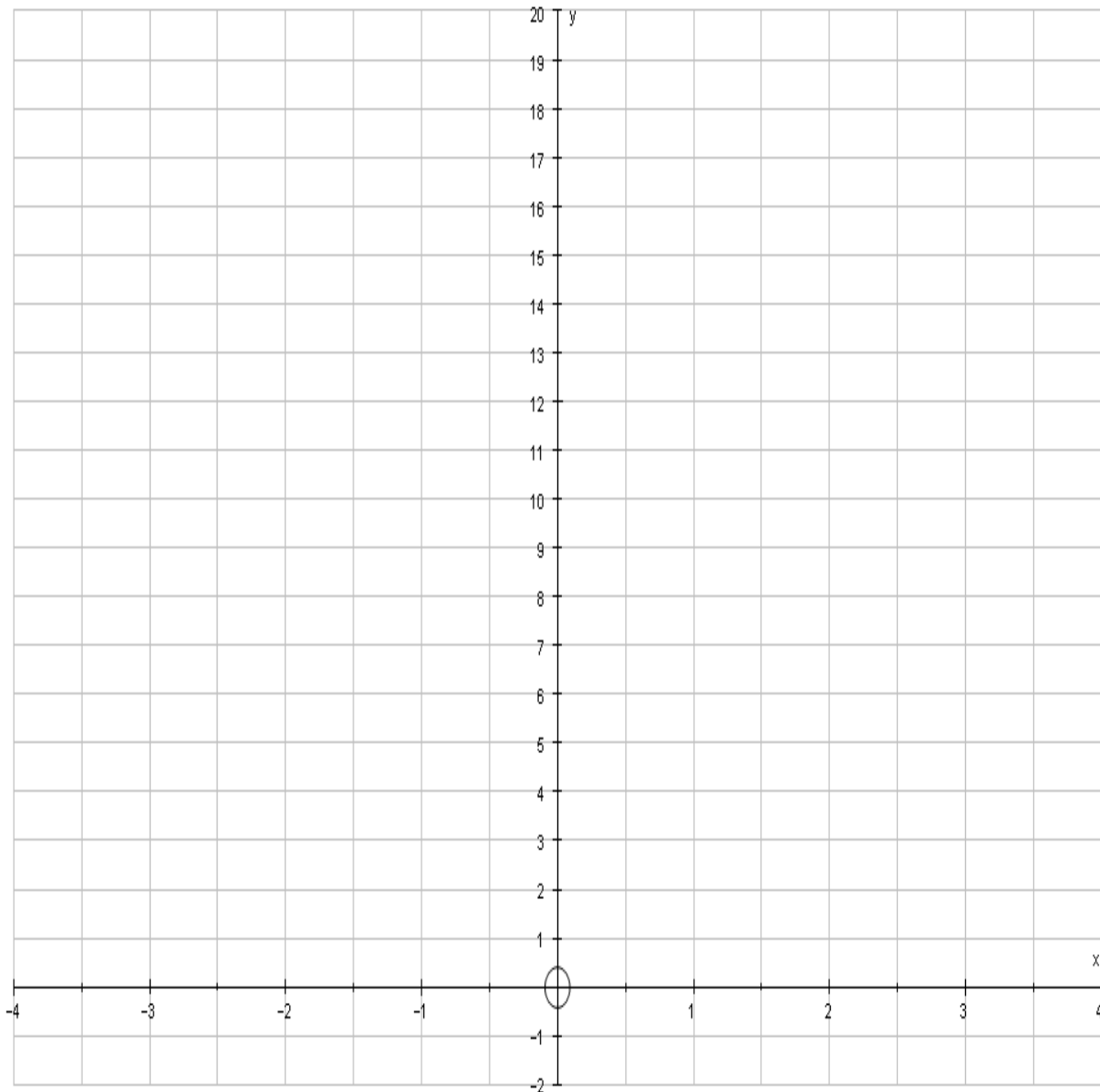
x	-4	-3	-2	-1	0	1	2	3
y	9							



$y = x^2$ has been translated horizontally left by 1 unit.

7. Sketch $y = (x - 2)^2$

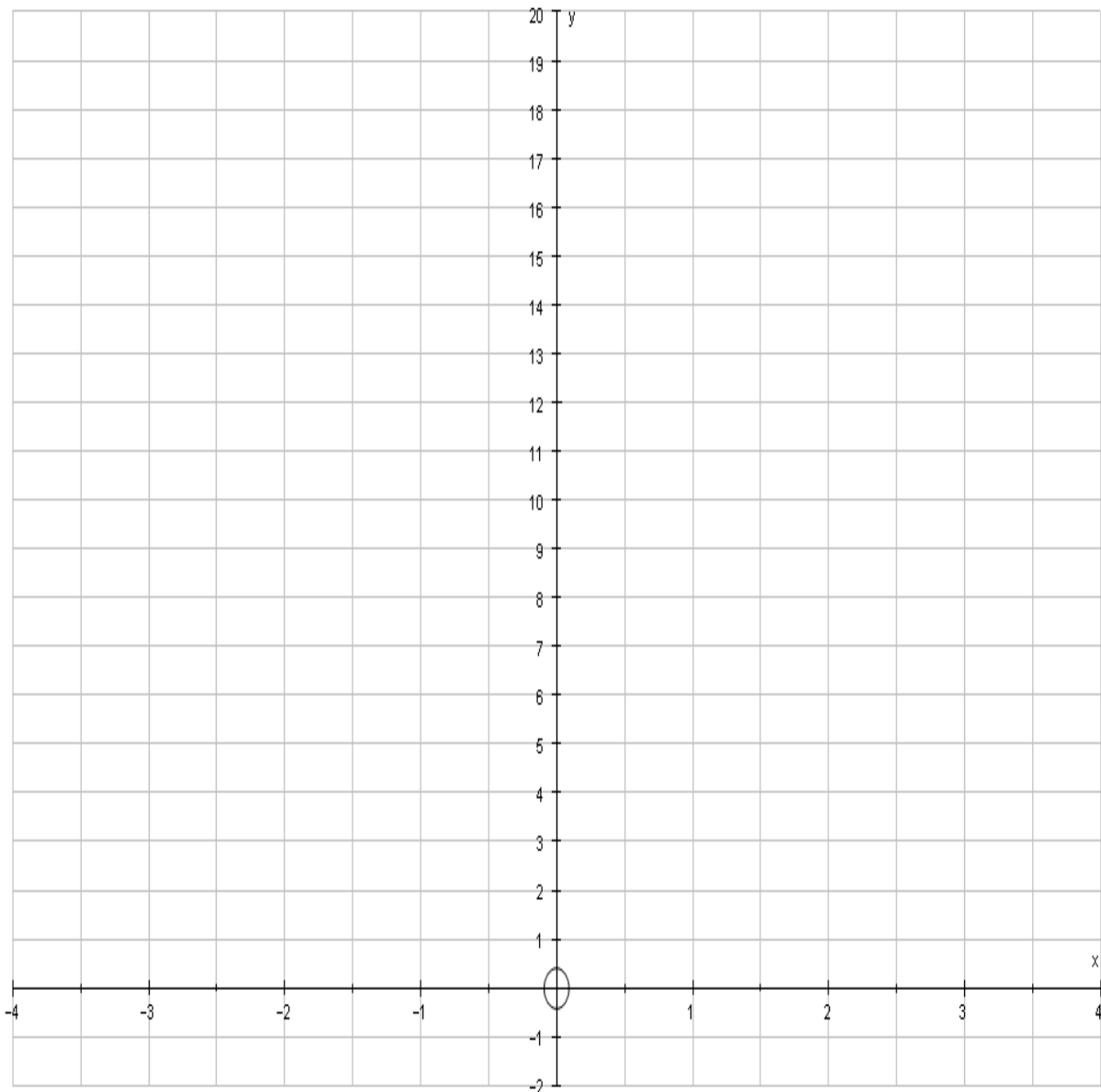
x	-2	-1	0	1	2	3	4
y							4



$y = x^2$ has been translated horizontally right by 2 units.

8. Sketch $y = (x + 1)^2 - 2$

x	-4	-3	-2	-1	0	1	2	3
y	7					2		



This combines horizontal and vertical translations of $y = x^2$

$$y = (x + 1)^2 - 2$$

1 to the left
2 down

Turning points for questions 1 - 8

If the co-efficient of x^2 is *positive*, the curve is a 'smile' with a minimum turning point.

If the coefficient of x^2 is *negative*, the curve is 'sad' with a maximum turning point.

- | | | |
|----|---------------------|---------------|
| 1. | $y = x^2$ | T.P. (0, 0) |
| 2. | $y = 2x^2$ | T.P. (0, 0) |
| 3. | $y = -x^2$ | T.P. (0, 0) |
| 4. | $y = x^2 + 2$ | T.P. (0, 2) |
| 5. | $y = x^2 - 1$ | T.P. (0, -1) |
| 6. | $y = (x + 1)^2$ | T.P. (-1, 0) |
| 7. | $y = (x - 2)^2$ | T.P. (2, 0) |
| 8. | $y = (x + 1)^2 - 2$ | T.P. (-1, -2) |

The axis of symmetry of each curve is a vertical line drawn through the T.P.

Here is what question 8 looks like:

