1. Introduction

Percent means
for each 100 or out of 100.

$$80\% = \frac{80}{100} = 80 \div 100 = 0.8$$

$$5\% = \frac{5}{100} = 5 \div 100 = 0.05$$

Practise by completing the following:

$$50\% = \frac{50}{100} =$$

$$20\% = \frac{20}{100} =$$

$$75\% =$$

$$125\% =$$

$$17.5\% =$$

2. Percentage change

a. A guitar has been reduced by 20% from its original price of £300. How much is it now selling for?

Method 1

$$\frac{20}{100} \times 300 = \frac{20 \times 300}{100} = \frac{20 \times 3}{1} = 60$$

£300 – £60 = **£240**

Method 2

The guitar is now worth 80% of its original price.

$$\frac{80}{100} \times 300 = \frac{80 \times 300}{100} = \frac{80 \times 3}{1} = \mathbf{240}$$

b. The value of a house increases by 5%. Calculate the new value if it had originally been £120 000.

Method 1

$$\frac{5}{100} \times 120000 = \frac{5 \times 120000}{100} = \frac{5 \times 1200}{1} = 6000$$

The change in value is £6000. As the value is increasing, we add:

£120 000 + £6000 = **£126 000**

Method 2

After the increase, the house is worth 105% of the original value.

$$\frac{105}{100} \times 120000 = \mathbf{126000}$$

c. A dress normally costing £70 was reduced by 30%. A blue cross sale takes off an extra 20%. What is the final cost of the dress?

We can do this in two stages:

$$\frac{70}{100} \times 70 = 0.7 \times 70 = 49$$

This new value is carried forward to the second reduction.

$$\frac{80}{100} \times 49 = 0.8 \times 49 = 39.2$$

The final cost of the dress is £$\mathbf{39.20}$

Is the final cost of the dress the same if you take off the original price 50% in one go? Explain your reasoning.

3. Appreciation

Appreciation means that the value is increasing. If you invest money you expect the amount to increase. Compound interest is added to the investment.

You can use a calculator for this type of question.

a. £1000 is invested at 5% per annum for 2 years. Calculate the amount of interest paid after 2 years. (Note: per annum or p.a. means 'each year'.)

1st year:

$$\frac{5}{100} \times 1000 = \frac{5000}{100} = 50$$

This interest is added to the original (or principal) amount.

End of Year 1 $\quad$ £1000 + £ 50 = £1050

This is carried forward (compounded) to the 2nd year.

2nd year:

$$\frac{5}{100} \times 1050 = \frac{5250}{100} = 52.5$$

This is added to the End of Year 1 amount.

End of Year 2 £1050 + £ 52.50 = £1102.50

The total interest gained is £1102.50 – £1000 = **£102.50**

b. Work out the value of the investment after 6 years.

Compound interest meant adding: 100% + 5% = 105% = 1.05 each year

Year 1: $1000 \times 1.05 = 1050$ *Year 2:* $1050 \times 1.05 = 1102.50$

This could be done in one step:

$$1000 \times 1.05 \times 1.05 = 1102.50$$

We could express the percentage multiplier as a power:

$$1000 \times 1.05^2 = 1102.50$$

For a 6 year investment this becomes:

$$1000 \times 1.05^6 = \mathbf{£1340.10}$$ (to the nearest penny)

4. Depreciation

This means that the value of a thing is decreasing. Economic downturn can cause house prices, share prices and fine art value to drop. Cars are a good example of depreciation.

1. The value of a car decreases by 35% in the first year and 12% in the second. Calculate the value after 2 years if it had been £25 000 new.

Method 1:

1st year:

$$\frac{35}{100} \times 25000 = 0.35 \times 25000 = 8750$$

As the value is depreciating, we subtract:

$$25000 - 8750 = 16250$$

2nd year:

$$\frac{12}{100} \times 16250 = 0.12 \times 16250 = 1950$$

As the value is depreciating, we subtract:

$$16250 - 1950 = 14300$$

So the value after 2 years is **£14300**

Method 2

A loss of 35% means we are left with 65%. So the multiplier is 0.65 for the 1st year and 0.88 for the 2nd year

$$25000 \times 0.65 \times 0.88 = 14300$$

2. A computer cost £899.99 new, exclusive of VAT. Its value depreciates by 40% p.a. for 3 years. If VAT (at a rate of 20%) had to be paid when new, calculate value of the computer after three years.

First calculate the cost inclusive of VAT:

$$\frac{120}{100} \times 899.99 = 1.2 \times 899.99 = 1079.99$$ (to the nearest penny)

A loss of 40% means we are left with 60% of the original which means the multiplier is 0.6 for the three years.

$$1079.99 \times 0.6^3 \ = 238.28$$ (to the nearest penny)

After three years, the value of the computer is £**238.28**

5. Mixing appreciation and depreciation

A house is bought for £225 000. In the first year its value rises by 8.5%. In the second year it drops by 5% and in the third it rises by 2.2%.

a. Calculate the value after three years.

$$225000 \times 1.085 \times 0.95 \times 1.022 = \mathbf{237020.96}$$ (to the nearest penny)

Appreciation 108.5%

Depreciation 95%

Appreciation 102.2%

The value after three years is £$\mathbf{237020.96}$ (to the nearest penny)

b. Express this as a % change of the original value.

$$\%\ change = \frac{change\ in\ value}{original\ value} \times 100$$

$$\frac{(237020.96-225000)}{225000} \times 100 = 5.3$$ (to 1 decimal place)

This represents a percentage increase of $\mathbf{5.3}$%

Exercise

1. A magazine has 4600 annual subscriptions. The owner wants that number to rise by 3% next year and 4.5% the year after.

   What is the target number of subscriptions after two years (rounded to nearest 50)?

2. A pay deal is settled as a 2.6% increase per annum, for three years. Bobbie's current annual salary is £27325.

   Calculate his salary in after three years, to the nearest pound.

3. The value of a £150000 house increases by 5% in the first year, 2.5% in the second year and 4% in the third.

   How much is it worth after three years?

4. A car was purchased for £15650. Two years later it was worth £9875.

   a. What is the current value as a percentage of the original purchase price? (Write your answer to 1 decimal place.)

   b. Work out the car's depreciation as a percentage of the purchase price. (Write your answer to 1 decimal place.)