When the coefficient of $x^2 = 1$

These are the most common type you'll be asked to factorise. It's important to first understand what's happening when multiplying out.

$$\begin{aligned}(x+5)(x+4) &= x^2 + 4x + 5x + 20 \\ &= x^2 + 9x + 20\end{aligned}$$

Both brackets had to start with x, as $x \times x = x^2$. The numbers in the brackets add to give 9 and multiply to give 20

How do we choose the correct pair of numbers?

Write down the factors of 20:

$20 = 20 \times 1 = 10 \times 2 = 5 \times 4$

Which have a sum of 9?

$20 + 1 = 21 \quad 10 + 2 = 12 \quad 5 + 4 = 9$ ✓

It has to be 5 and 4:

$x^2 + 9x + 20 = (x+5)(x+4) \quad [or\ (x+4)(x+5)]$

Always check by multiplying out.

Now try: $y^2 + 3y - 18$

Two factor pairs of -18 are -6×3 and 6×-3 (There are others too.)

We now have to consider if the pair of factors can *add* to give 3.

$(-6) + (+3) = -3$ ✗ $\quad (+6) + (-3) = 3$ ✓

$y^2 + 3y - 18 = (y+6)(y-3)$

Negative signs are a common source of mistake, be careful.

Check you understand this one:

$y^2 - 3y - 18 = (y-6)(y+3)$

Now try: $x^2 - 11x + 24$

Two factor pairs of 24 are $(-8) \times (-3)$ and $(+8) \times (+3)$

$(-8) + (-3) = -11$ ✓

$$x^2 - 11x + 24 = (x - 8)(x - 3)$$

Exercise A

Factorise these and check your solutions by multiplying the brackets.

1. $a^2 + 3a + 2$
2. $b^2 + 4b + 4$
3. $c^2 - 2c - 3$
4. $d^2 + 4d - 21$
5. $e^2 - 10e + 16$
6. $f^2 - 6f + 9$

When the coefficient of $x^2 \neq 1$

1. $3x^2 + 7x + 2$

We must include factors of 3 and 2 which must add to give 7

$3 = 1 \times 3$

$2 = 2 \times 1$ gives $(2 \times 1) + (3 \times 1) = 5$

$3 = 1 \times 3$

$2 = 2 \times 1$ gives $(1 \times 1) + (3 \times 2) = 7$ ✓

So we need to match the 3 with 2 and 1 with 1:

$$3x^2 + 7x + 2 = (3x + 1)(x + 2)$$

2. $5a^2 - 2a - 7$

$\pm 5 \times \pm 1$

$\pm 7 \times \pm 1 \rightarrow \pm 35 \pm 1$

$\pm 5 \times \pm 1$

$\pm 7 \times \pm 1 \rightarrow \pm 7 \pm 5 = -2$ ✓

$$5a^2 - 2a - 7 = (5a - 7)(a + 1)$$

3. $7x^2 + 6x - 13$

$7 = (+7) \times (+1)\ or\ (-7) \times (-1)$

$-13 = (-13)\ \times (+1)\ or\ (+13) \times (-1)$

Signs: the middle term is + so make the larger number positive.

There must be one – sign.

$$7x^2 + 6x - 13 = (7x + 13\ )(x - 1)$$

Always multiply out to check.

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4. Try this one where the coefficient of y^2 isn't prime.

$6y^2 - 27y + 12$

There appear to be two possible solutions:

$(6y - 3)(y - 4) = 6y^2 - 27y + 12$

$(3y - 12)(2y - 1) = 6y^2 - 27y + 12$

Look carefully and you'll see that these two solutions are identical:

$(6y - 3)(y - 4) = 3(2y - 1)(y - 4)$

$(3y - 12)(2y - 1) = 3(y - 4)(2y - 1)$

$3(y - 4)(2y - 1) = 3(2y - 1)(y - 4)$

Always look for simple common factors first.

$6y^2 - 27y + 12 = 3(2y^2 - 9y + 4)$

$\qquad\qquad\qquad = 3(2y - 1)(y - 4)$

Exercise B

Identify the simple common factor then factorise completely:

1. $3a^2 + 9a + 6$
2. $5b^2 + 20b + 20$
3. $\frac{1}{2}c^2 - c - \frac{3}{2}$
4. $3d^2 + 12d - 63$

The difference of two squares

This is a special case of double bracket factorising:

$$a^2 - b^2 = (a + b)(a - b)$$

Check this in reverse:

$$(a + b)(a - b) = a^2 - ab + ba - b^2$$
$$= a^2 - ab + ab - b^2$$
$$= a^2 - b^2$$

The middle terms cancel out.

The trick is to spot square numbers.

$$x^2 - 16 = x^2 - 4^2$$
$$= (x - 4)(x + 4)$$
$$49 - y^2 = 7^2 - y^2$$
$$= (7 + y)(7 - y)$$
$$4x^2 - 81y^2 = (2x)^2 - (9y)^2$$
$$= (2x - 9y)(2x + 9y)$$

Always look for common factors:

$$27x^2 - 75y^2 = 3[9x^2 - 25y^2]$$
$$= 3[(3x + 5y)(3x - 5y)]$$
$$128a^2 - 2 = 2[64a^2 - 1]$$
$$= 2[(8a)^2 - 1^2]$$
$$= 2(8a - 1)(8a + 1)$$

Finally, try this exam question.

Factorise $9.3^2 - 0.7^2$