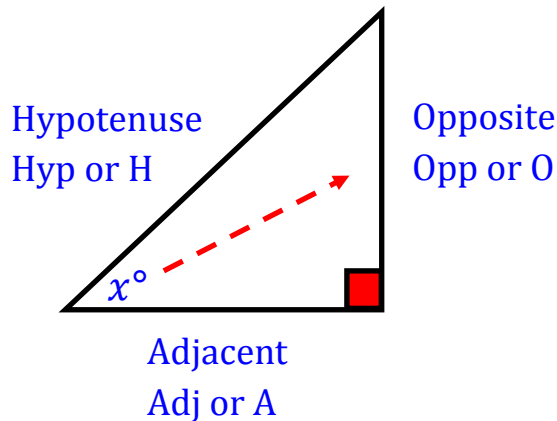


Basic trigonometry

Pythagoras relates the lengths of **sides** in a right-angled triangle.

Trigonometry allows us to calculate sides and **angles**.

Drawing a sketch of the triangle and labelling the sides is crucial.

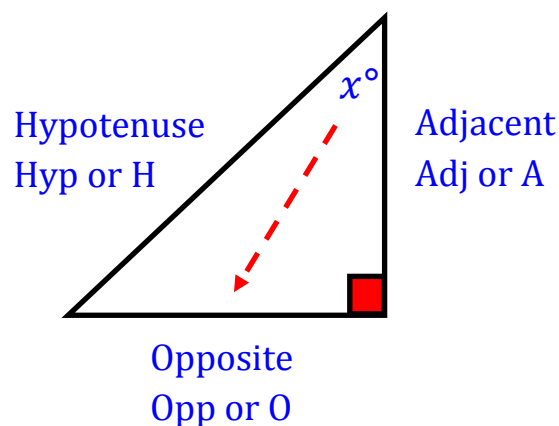


The **hypotenuse (hyp)** is always opposite the right angle.

Adjacent (adj) means 'next to' and is the side that touches the angle we're interested in.

Opposite is the side that doesn't touch the angle of interest. It is opposite the angle.

If we were interested in the other angle, we would have to label accordingly and the adj and opp would swap over:



We define the Sine (sin), Cosine (cos) and Tangent (tan) functions as ratios of two of the sides:

Memorise:

$$\sin x = \frac{\text{opp}}{\text{hyp}}$$

$$\cos x = \frac{\text{adj}}{\text{hyp}}$$

$$\tan x = \frac{\text{opp}}{\text{adj}}$$

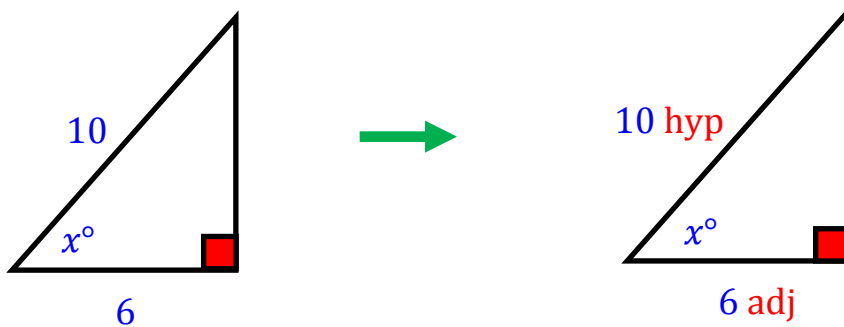
A commonly used mnemonic is:

SOH CAH TOA

We can use these definitions to solve for unknown angles or sides as long as we have two pieces of information.

Working out an angle

1.



Always do a sketch and mark on the sides relative to the angle.

Write down

SOH **CAH** TOA

Here we know the adjacent and the hypotenuse and want to find an **angle** so we use **Cosine**.

$$\cos x = \frac{\text{adj}}{\text{hyp}} \rightarrow \cos x = \frac{6}{10} \rightarrow \cos x = 0.6$$

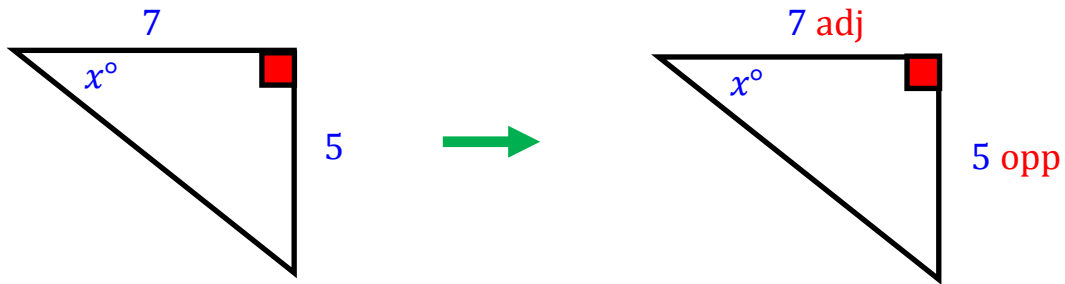
We now apply the inverse cosine (\cos^{-1}) function to calculate x .

On your calculator this is activated by pressing **SHIFT** or **2nd function** then **cos** then finally **=**

$$\cos x = 0.6 \rightarrow x = \cos^{-1}(0.6) \rightarrow x = 53.1^\circ$$

Make sure your calculator is set to **degree mode**.

2.



Always do a sketch and mark on the sides relative to the angle.

Write down

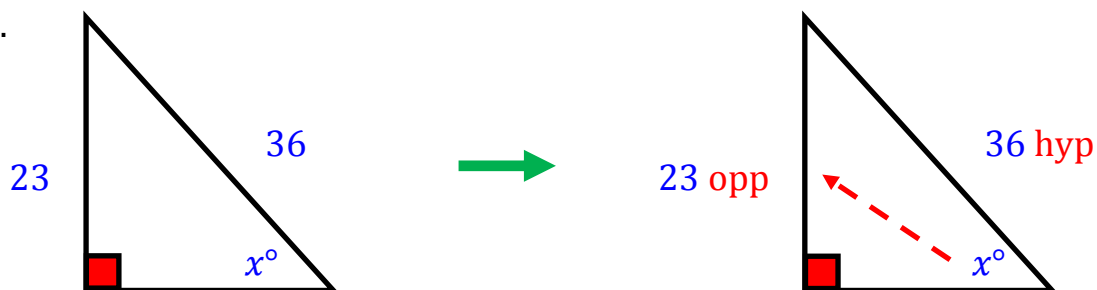
SOH CAH TOA

Here we know the adjacent and the opposite so we use **Tangent**.

$$\tan x = \frac{\text{opp}}{\text{adj}} \rightarrow \tan x = \frac{5}{7}$$

$$x = \tan^{-1}\left(\frac{5}{7}\right) \rightarrow x = 35.5^\circ$$

3.



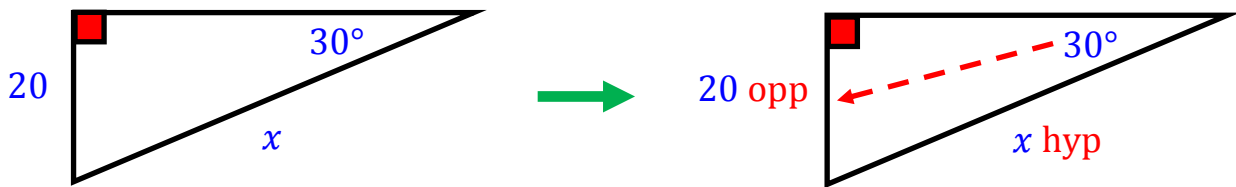
SOH CAH TOA

Here we know the opposite and hypotenuse so we use **Sine**.

$$\sin x = \frac{\text{opp}}{\text{hyp}} \rightarrow \sin x = \frac{23}{36}$$

$$x = \sin^{-1}\left(\frac{23}{36}\right) \rightarrow x = 39.7^\circ$$

Working out the length of a side



SOH CAH TOA

We want the hypotenuse and are given the opposite, so we use **Sine**.

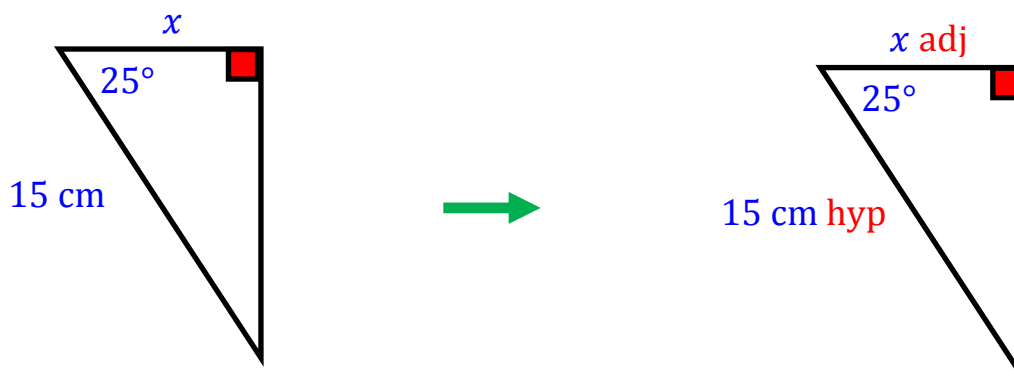
$$\sin x = \frac{\text{opp}}{\text{hyp}} \rightarrow \sin 30^\circ = \frac{20}{x}$$

$$\rightarrow x \sin 30^\circ = 20 \rightarrow x = \frac{20}{\sin 30^\circ} = 40$$

Multiply both sides by x to move it over to the LHS.

Divide both sides by $\sin 30^\circ$ to leave x on its own.

5.



SOH CAH TOA

It is the Adjacent we want and the hypotenuse is given, so we use **Cosine**.

$$\cos x = \frac{\text{adj}}{\text{hyp}} \rightarrow \cos 25^\circ = \frac{x}{15}$$

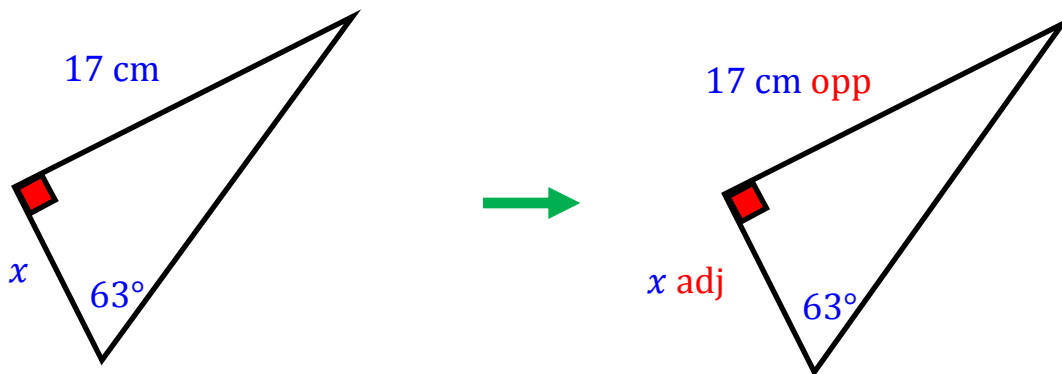
$$x = 15 \cos 25^\circ = 13.6 \text{ cm}$$

Multiply both sides by 15 to move it over to the LHS and leave x on its own.

Remember that the hypotenuse is always the longest side. This can be a useful check that your answer seems reasonable.

Adj = 13.6 cm Hyp = 15 cm ✓

6.

SOH CAH **TOA**

It is the Adjacent we want and the opposite is given, so we use **Tangent**.

$$\tan x = \frac{\text{opp}}{\text{adj}} \rightarrow \tan 63^\circ = \frac{17}{x}$$

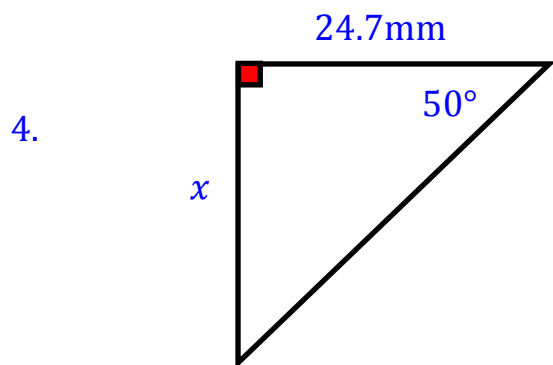
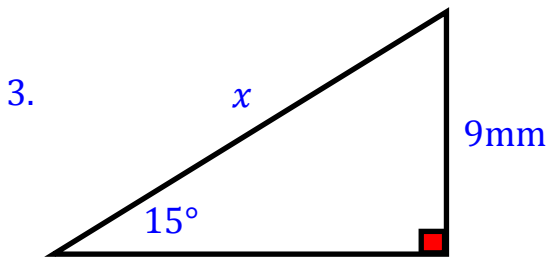
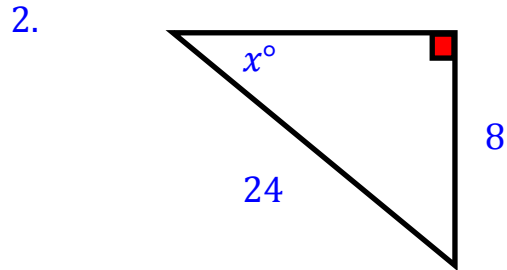
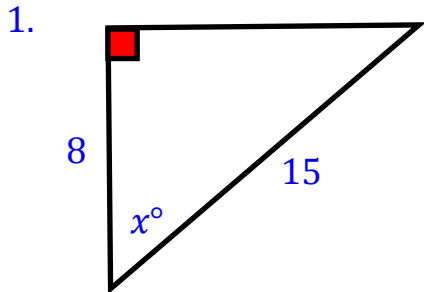
$$x \tan 63^\circ = 17 \rightarrow x = \frac{17}{\tan 63^\circ} = 8.7 \text{ cm}$$

Multiply both sides by x to move it over to the LHS.

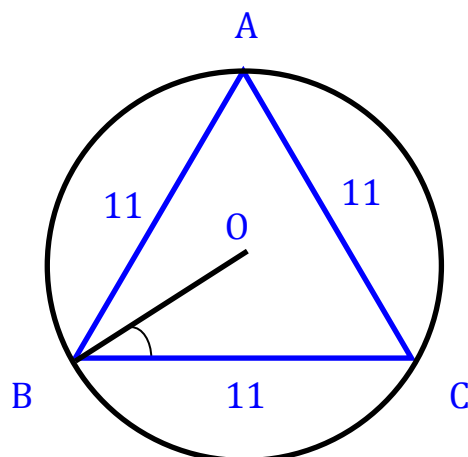
Divide both sides by $\tan 63^\circ$ to leave x on its own.

Exercise

Work out the value of x :



5. Triangle ABC is equilateral with sides 11 cm long, as shown below.
- (a) Write down the size of angle OBC.
- (b) Calculate the length of the radius OB.

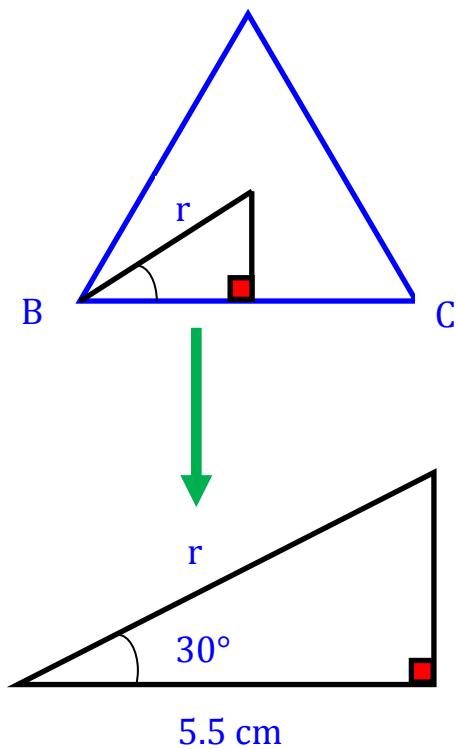


Solutions

1. 57.8°
2. 19.5°
3. 34.8 m
4. 29.4 m

5a. Each angle in an equilateral triangle is 60° . The line OB cuts this in half, so angle $OBC = 30^\circ$.

b. To apply trigonometry we need to create a right angled triangle (base = $11 \div 2 = 5.5$ cm).



SOH **CAH** TOA

$$\cos x = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 30^\circ = \frac{5.5}{r}$$

$$r = \frac{5.5}{\cos 30^\circ}$$

$$r = 6.35 \text{ cm}$$