A few years ago, in the town of Winchester in Southern England, the manager of a car showroom came up with an idea which quite caught the imagination of the public. He advertised a simple competition whereby people paid 50p, then rolled seven dice. If each die showed a six, the person won a luxury car (valued then at £35 000).

Your job is to analyse, from a probabilistic point of view, whether this was a 'fair' game or not.

Organising your ideas

What questions do you need to ask to help you answer the question in bold above?
(Write them – and the answers – in this space)

Calculations

What is the probability of rolling 7 sixes when you roll 7 dice?

How many times would you expect to have to roll 7 die before you get 7 sixes?

How much money would you expect to pay (50p per roll) before you get 7 sixes?

Conclusion(s)

Was the game fair?

If ‘yes’ – explain/justify:

If ‘no’ – explain why it wasn’t fair and suggest as many ways as you can in which the game could be made fairer.

What similar games/competitions have you heard about? Were they fair or not? Explain.

What games could you run at school as a fund-raiser that would be fair (yet likely to be profitable?)

Teacher notes

Although originally designed for use with the IB programme, this worksheet works equally well for GCSE or AS level. It can be easily adapted (different currencies, different games etc.), or extended to include such things as expected values.

Solutions to calculations and guide to discussions

Preliminary discussion could centre on independent events and how we handle the probability of events A and B both happening. There could be discussions around what a fair game is.

Probability of throwing 7 sixes with 7 dice $= \left(\frac{1}{6}\right)^7 = \frac{1}{279\,936} \approx \frac{1}{280\,000}$

This means that you would have to roll the dice about $280\,000$ times, before you see 7 sixes.

(But – be aware that this *doesn't* mean it will happen once in $280\,000$ throws - it ***could*** happen on the first throw, it may **not** happen in a million throws. It means that, if millions and millions of people performed the experiment, over millions of years, on average you would see this result once in $280\,000$ throws).

This being the case, the expected income before the car is won is $£0.50 \text{ x } 280\,000 = £140\,000$

If the showroom manager received $£140\,000$ before the $£35\,000$ car was won, this would suggest that the game isn't fair – after all, he would be making a 300% profit! Compare this to the kind of percentage profits that gaming machines etc. generate.

That said – the manager may well talk persuasively about risk. He might say that he is taking a huge risk in allowing this game to be played (the car could be won on the first throw, in which case the manager loses $£34\,999.50$). Students could work out just how small this risk is.

In terms of making the game fairer – many students will suggest trying 6 dice instead of 7, and having the same rules. Interestingly this doesn't work – the manager would stand to make a loss. Students might then suggest ideas such as:

- allowing more than one roll of the dice
- bringing the price of participation down (to, say 20p)
- changing the target to 7 sixes or 7 ones etc.

They should be able to back up these ideas with appropriate calculations.

Other things to consider and discuss:

1. What is the population of Winchester?
2. How many people would be likely to take part in this trial?
3. How valuable was the (free) publicity to the showroom manager?
4. What is a 'fair' or 'reasonable' profit in business?