A. Right angled triangle trigonometry

Just like using Pythagoras' trigonometry needs a right angled triangle.

We use Pythagoras' rule to find a missing side length in a right angled triangle when we know the other two side lengths.

We use trigonometry when an angle is involved in the question.

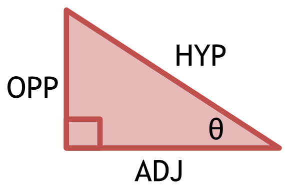
Trigonometry uses 3 formulae linking an angle and two sides in a right angled triangle. Providing we know any 2 terms in one of the formula we can calculate the 3rd term.

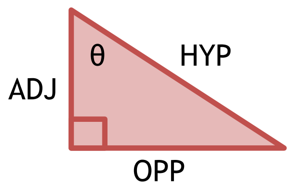
The formulae use **SINE, COSINE** or **TANGENT**
and the names of two of the sides in the triangle.

OPPosite is opposite the angle you are using

HYPotenuse is opposite the right angle

ADJacent is next to the angle being used





The formulae you need to learn are:

$$\text{Sin } \theta = \frac{\text{OPP}}{\text{HYP}} \qquad \text{Cos } \theta = \frac{\text{ADJ}}{\text{HYP}} \qquad \text{Tan } \theta = \frac{\text{OPP}}{\text{ADJ}}$$

SOH CAH TOA may help you remember the formulae

Method

1. Identify your right angled triangle
2. Label the sides OPP, ADJ and HYP (this is dependent on the angle you are using)
3. Decide whether to use SIN θ, COS θ or TAN θ

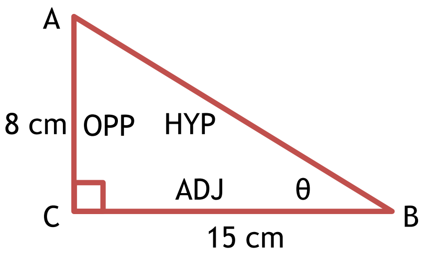
   You must use a formula that has the term you need to find and you must know the other two values – (You can't work with 2 unknown values)
4. Write down your chosen formula and fill in the known information
5. Use one of the 3 methods to find your unknown value

B. The 3 methods

There are only 3 methods to learn: one to find an angle, one to find a numerator and one to find a denominator.

1. Finding an unknown angle

Find angle ABC



Use the inverse function

$\text{Tan } \theta = \frac{\text{OPP}}{\text{ADJ}}$

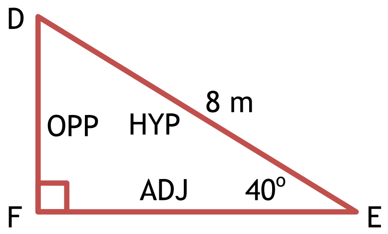
$\text{Tan B} = \frac{8}{15}$

$\text{B} = \text{Tan}^{-1}\left(\frac{8}{15}\right)$

B = 28.1° (1 dp)

2. Finding an unknown numerator

Find length DF



$\text{Sin } \theta = \frac{\text{OPP}}{\text{HYP}}$

$\text{Sin } 40 = \frac{\text{DF}}{8}$

8 x Sin 40 = DF

DF = 5.1 m (1 dp)

3. Finding an unknown denominator

Find length GH

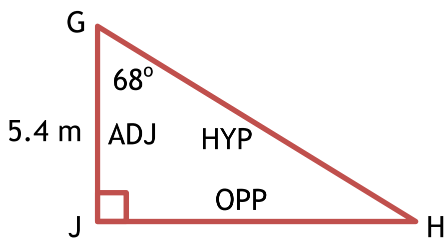
Swap places

$\text{Cos } \theta = \frac{\text{ADJ}}{\text{HYP}}$

$\text{Cos } 68 = \frac{5.4}{\text{GH}}$

$\text{GH} = \frac{5.4}{\text{Cos } 68}$

GH = 14.4 m (1 dp)

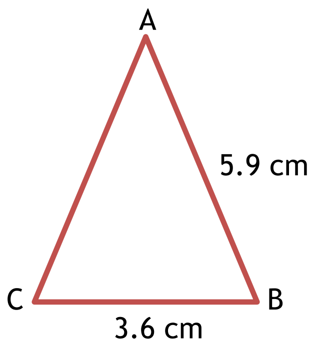


It will have to be one of these 3 methods.
Learn them!

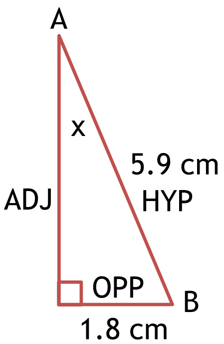
C. Harder examples

Sometimes it is harder to find the right angled triangle needed and sometimes there will be more than one process needed in the same question.

1. ABC is an isosceles triangle. Find the size of angle CAB.



An isosceles triangle has a line of symmetry that cuts it into two identical right angled triangles



Sin θ = $\frac{\text{OPP}}{\text{HYP}}$

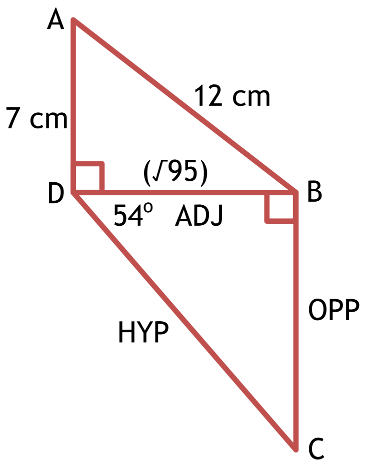
Sin x = $\frac{1.8}{5.9}$

x = $\text{Sin}^{-1}\left(\frac{1.8}{5.9}\right)$ = 17.7632 ...

Angle CAB = 2 x 17.7632 ...

Angle CAB = 35.5° (1 dp)

2. The diagram shows two right angled triangles. Find the area of triangle DBC.



12^2 = 144 7^2 = 49

144 - 49 = 95

DB = √95 (DB = 9.746794345)

Tan θ = $\frac{\text{OPP}}{\text{ADJ}}$

Tan 54 = $\frac{\text{BC}}{\sqrt{95}}$

To keep accuracy use full calculator values so you only round your answer once at the end

BC = √95 x Tan 54 (BC = 13.41531152)

Find DB using Pythagoras then find BC using trigonometry. Now we can find the area.

Area = $\frac{1}{2}$ x DB x BC

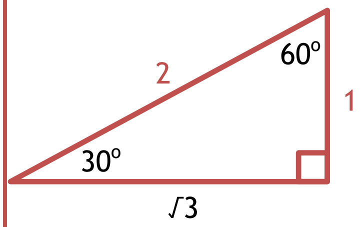
Area = $\frac{1}{2}$ x √95 x √95 x Tan 54

Area = 65.4 cm² (1 dp)

D. Exact values from the right angled triangles 30, 60, 90 and 45, 45, 90

Learn Sin 30° = $\frac{1}{2}$ and Tan 45° = 1

Sin 30° = $\frac{1}{2}$ $\frac{OPP}{HYP}$ = $\frac{1}{2}$ ∴ OPP = 1 HYP = 2



OPP, ADJ and HYP are dependent on the angle being used

Using Pythagoras we can calculate the third side to be √3

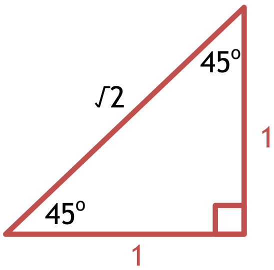
We now know exact values for:

Sin 30° = $\frac{1}{2}$ Cos 60° = $\frac{1}{2}$

Cos 30° = $\frac{\sqrt{3}}{2}$ Sin 60° = $\frac{\sqrt{3}}{2}$

Tan 30° = $\frac{1}{\sqrt{3}}$ Tan 60° = $\frac{\sqrt{3}}{1}$ = √3

Tan 45 ° = 1 $\frac{OPP}{ADJ}$ = $\frac{1}{1}$ ∴ OPP = 1 ADJ = 1



Using Pythagoras we can calculate the third side to be √2

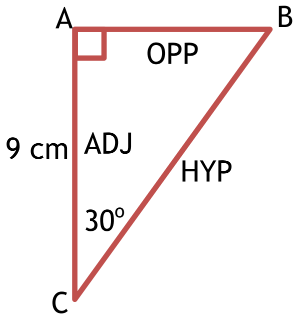
We now know exact values for:

Tan 45° = $\frac{1}{1}$ = 1

Sin 45° = $\frac{1}{\sqrt{2}}$

Cos 45° = $\frac{1}{\sqrt{2}}$

1. Without using a calculator find the exact length of AB



Rationalise the denominator

Tan θ = $\frac{OPP}{ADJ}$ Tan 30 = $\frac{AB}{9}$

$\frac{1}{\sqrt{3}} = \frac{AB}{9}$

$\frac{9}{\sqrt{3}}$ = AB

AB = $\frac{9}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{3}$ = **3√3 cm**

2. Without using a calculator find the exact value of sin 60° x sin 45°

$\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{4}$

3. Without using a calculator find the exact value of sin 60° - tan 30°

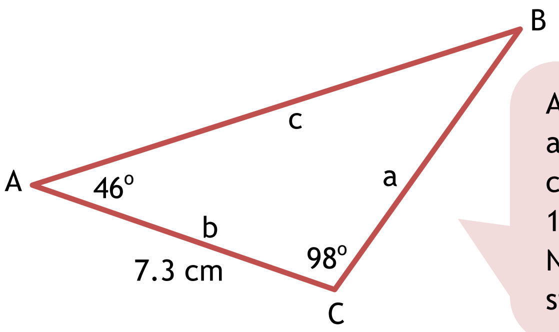
$\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{3}} = \frac{\sqrt{3}\sqrt{3} - 2}{2\sqrt{3}} = \frac{3 - 2}{2\sqrt{3}} = \frac{1}{2\sqrt{3}} = \frac{1\sqrt{3}}{2\sqrt{3}\sqrt{3}} = \frac{\sqrt{3}}{6}$

E. The sine and cosine rule for non-right angled triangles

The sine rule	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ Lengths on top to find a length	$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ Angles on top to find an angle
The cosine rule	$a^2 = b^2 + c^2 - 2bc\cos A$	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

If you can partner one length with its opposite angle you are likely to need the sine rule.

1. Find the length AB.



As we know two angles we can calculate the 3rd. 180 - 98 - 46 = 36°
Now we can partner side b with angle B

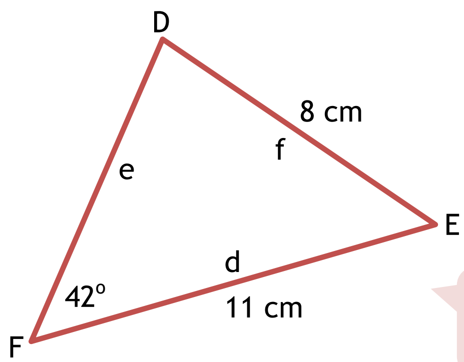
$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\frac{AB}{\sin 98} = \frac{7.3}{\sin 36}$$

$$AB = \frac{7.3 \times \sin 98}{\sin 36}$$

AB = 12.3 cm (1 dp)

2. Find the size of angle FDE.



We can partner side f with angle F

$$\frac{\sin D}{d} = \frac{\sin F}{f}$$

$$\frac{\sin D}{11} = \frac{\sin 42}{8}$$

$$\sin D = \frac{11 \times \sin 42}{8}$$

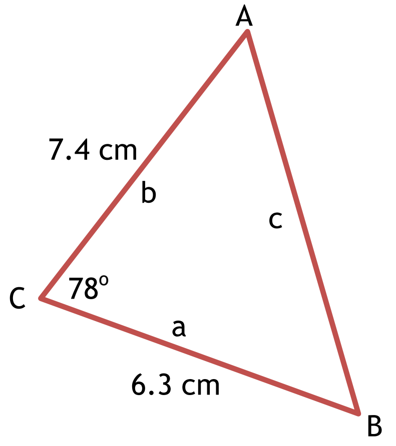
$$D = \sin^{-1}\left(\frac{11 \times \sin 42}{8}\right)$$

FDE = 66.9° (1 dp)

If we know all three side lengths we can use the cosine rule to find an angle.

If we have two sides and the angle between them we can use the cosine rule to find the 3rd side.

3. Find length AB giving your answer to 2 significant figures



$c^2 = a^2 + b^2 - 2ab\cos C$

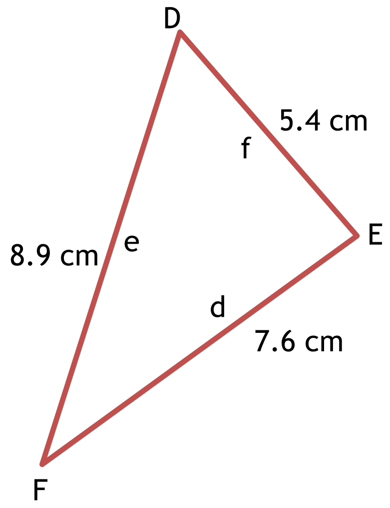
$c^2 = 6.3^2 + 7.4^2 - (2 \times 6.3 \times 7.4 \times \cos 78)$

$c^2 = 75.06431395$

$c = \sqrt{75.06431395}$

AB = 8.7 cm (2 sf)

4. Find the size of angle DEF giving your answer to 2 significant figures



$e^2 = d^2 + f^2 - 2df\cos E$

$\cos E = \frac{d^2 + f^2 - e^2}{2df}$

$\cos E = \frac{7.6^2 + 5.4^2 - 8.9^2}{2 \times 7.6 \times 5.4}$

$E = \cos^{-1}\left(\frac{7.6^2 + 5.4^2 - 8.9^2}{2 \times 7.6 \times 5.4}\right)$

DEF = 85° (2 sf)

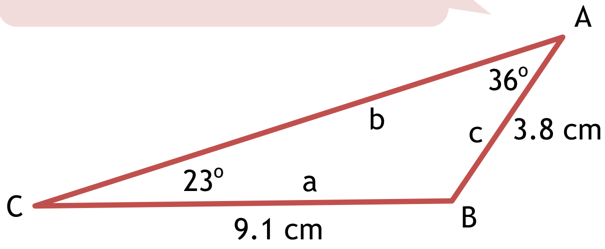
F. Area of a non-right angled triangle

When we know two sides and the angle between them we can find the area of a triangle using

$$\textbf{Area} = \frac{1}{2}ab\sin C$$

1. Find the area of triangle ABC giving your answer to 3 significant figures

We can calculate the angle between the two known sides



Angle B = 180 - 23 - 36 = 121°

$\text{Area} = \frac{1}{2}ac\sin B$

$\text{Area} = \frac{1}{2} \times 9.1 \times 3.8 \times \sin 121$

Area = 14.8 cm² (3 sf)