

One function leading to six A2 topics - Partial fractions, differentiation, integration, use of logarithms, Binomial expansion and approximations

$$f(x) \equiv \frac{2 - 9x}{(x - 3)(2x - 1)^2}$$

1. Express $f(x)$ in partial fractions
2. Find $f'(x)$
3. Evaluate $\int_1^2 f(x)dx$ giving you answer as $\frac{a}{b} + \ln(c)$, where a , b and c are integers
4. Find the first 4 terms in the Binomial expansion of each partial fraction
5. Estimate the value of $f(x)$ when $x = 0.001$ using the first 3 terms in the expansion of $f(x)$
6. Evaluate $f(x)$ directly to 6 decimal places when $x = 0.001$

Teacher notes

$$f(x) \equiv \frac{2 - 9x}{(x - 3)(2x - 1)^2}$$

$$1. f(x) \equiv \frac{2}{(2x-1)} + \frac{1}{(2x-1)^2} - \frac{1}{(x-3)}$$

$$2. f'(x) = -\frac{4}{(2x-1)^2} - \frac{4}{(2x-1)^3} + \frac{1}{(x-3)^2} \equiv \frac{36x^2 - 66x - 1}{(x-3)^2(2x-1)^3}$$

$$3. \int_1^2 f(x) dx = \frac{1}{3} + \ln(6) \quad \left[\ln(2x - 1) - \frac{1}{2(2x-1)} - \ln(x - 3) + c \right]$$

$$4. \frac{2}{(2x-1)} = -2 - 4x - 8x^2 - 16x^3 - \dots \quad \frac{1}{(2x-1)^2} = 1 + 4x + 12x^2 + 32x^3 + \dots$$

$$\frac{1}{(x-3)} = -\frac{1}{3} - \frac{x}{9} - \frac{x^2}{27} - \frac{x^3}{81} - \dots$$

$$5. f(x) \approx -\frac{2}{3} + \frac{x}{9} + \frac{109x^2}{27} = -0.666552 \text{ when } x = 0.001$$

$$6. \text{ The exact value of } f(x) = -0.666552 \text{ when } x = 0.001$$