One parametric function leading to many A2 topics – Curve sketching, range and domain, differentiation, area and volume by integration, trapezium rule, percentage error, trigonometric equations and use of identities

$$x = 3\cos(t)\,, y = \sin(t)\,, 0 \leq t \leq \pi$$

1. Complete the table below, with entries to 1 decimal place where appropriate

t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
x		2.1			
y				0.7	

2. Sketch the curve

3. Write down the domain and range

4. Use integration on the parametric function to find the exact area enclosed between the curve and the x-axis

5. Find the Cartesian equation of the curve giving your answer in the form $y = f(x)$

6. Use the trapezium rule with 6 trapeziums to approximate the area, to 4 decimal places, enclosed between the curve and the x-axis

7. Calculate the percentage error, to 4 decimal places, between the exact area and the approximate area

8. Why does increasing the number of trapeziums improve the approximation for the area?

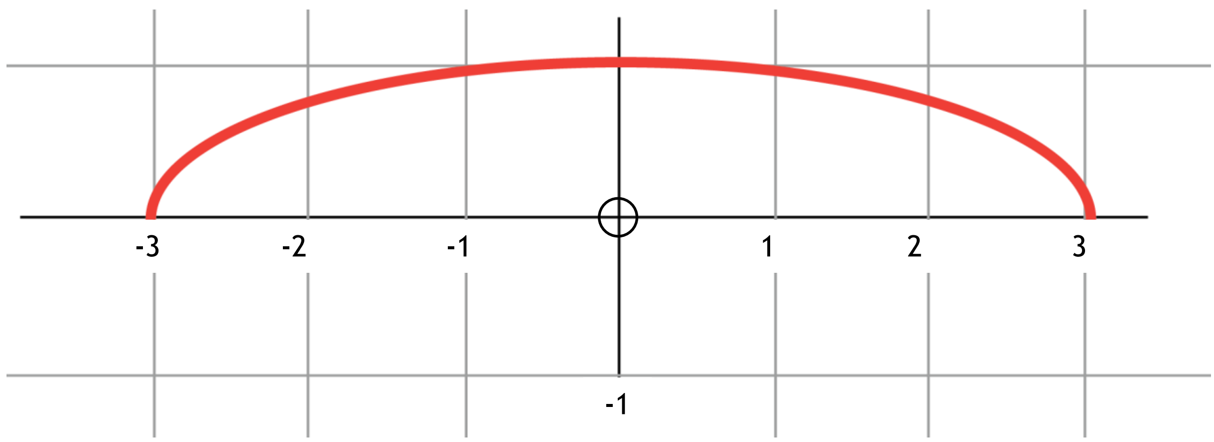
9. Use integration of parametric functions to find the exact volume when the area between the curve and the x-axis is rotated 2π radians about the x-axis

Notes and answers

1.

t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
x	3	2.1	0	-2.1	-3
y	0	0.7	1	0.7	0

2.



3. Domain $-3 \leq x \leq 3$ Range $0 \leq y \leq 1$

4. When $x = -3, t = 0$ and when $x = 3, t = \pi$, so as t increases the curve is drawn in a clockwise direction and hence $\int_0^\pi y \frac{dx}{dt} dt = 3\int_0^\pi sin^2(t)\, dt$ will give a positive value and the area is $\frac{3\pi}{2}$ units squared. The curve is half the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with area equal to $\pi(a)(b)$. This area is $\frac{1}{2}\pi(3)(1) = \frac{3\pi}{2}$

5. $y = \frac{1}{3}\sqrt{9 - x^2}$

6.

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{3}$	$\frac{\sqrt{5}}{3}$	$\frac{\sqrt{8}}{3}$	1	$\frac{\sqrt{8}}{3}$	$\frac{\sqrt{5}}{3}$	$\frac{1}{3}$

Area $\approx$ 4.7097

7. Percentage error $\approx$ 0.0578%

8. Increasing the number of trapeziums, reduces the gap between the top of each trapezium and the curve

9. $V = \pi\int_0^\pi y^2 \frac{dx}{dt} dt = 3\pi\int_0^\pi sin^3(t)dt = 4\pi$ units cubed