Iterations

1. Using: $x_{n+1} = 5 - 2x_n$ with $x_0 = 2$, find the value of x_3.

2. Using: $x_{n+1} = 2 + \frac{2}{x_n}$ with $x_0 = 1$, find the value of x_4.

3. Using: $x_{n+1} = 8 - \frac{5}{x_n^2}$ with $x_0 = 1$, find the value of x_4.

4. Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{8}{9} - \frac{x_n^3}{9}$ to find the value of x_3.

5. Starting with $x_0 = 3$, use the iteration formula $x_{n+1} = \sqrt[3]{3x_n + 11}$ to find the value of x_3.

6. Starting with $x_0 = 3$, use the iteration formula $x_{n+1} = \sqrt{5 - \frac{1}{x_n}}$ to find the value of x_3.

Rearranging and iterations

1a. Show that $x^2 - 4x - 9 = 0$ can be written in the form $x = \sqrt{9 + 4x}$.

b. Use the iteration formula $x_{n+1} = \sqrt{9 + 4x_n}$ to find x_3 to 2 decimal places. Start with $x_0 = 2$.

2a. Show that $x^3 - x - 19 = 0$ can be written in the form $x = \sqrt[3]{x + 19}$.

b. Use the iteration formula $x_{n+1} = \sqrt[3]{x_n + 19}$ to find x_4 to 2 decimal places. Start with $x_0 = 0$.

3a. Show that $x^2 + 4x - 29 = 0$ can be written in the form $x = \sqrt{29 - 4x}$.

b. Use the iteration formula $x_{n+1} = \sqrt{29 - 4x_n}$ to find x_3 to 2 decimal places. Start with $x_0 = 1$.

4a. Show that $x^3 + 3x - 260 = 0$ can be written in the form $x = \sqrt[3]{260 - 3x}$.

b. Use the iteration formula $x_{n+1} = \sqrt[3]{260 - 3x_n}$ to find x_4 to 2 decimal places. Start with $x_0 = 2$.

5a. Show that $2x^2 + 3x - 6 = 0$ can be written in the form $x = \frac{6-2x^2}{3}$.

b. Use the iteration formula $x_{n+1} = \frac{6-2x_n{}^2}{3}$ to find x_4 to 2 decimal places. Start with $x_0 = 1$.

6a. Show that $\frac{x^4}{2} - 3x = 0$ can be written in the form $x = \sqrt[4]{6x}$.

b. Use the iteration formula $x_{n+1} = \sqrt[4]{6x_n}$ to find x_4 to 2 decimal places. Start with $x_0 = 4$.

Answers

1. $x_1 = 1$
   $x_2 = 3$
   $x_3 = -1$

2. $x_1 = 4$
   $x_2 = 2.5$
   $x_3 = 2.8$
   $x_4 = 2.714...$

3. $x_1 = 3$
   $x_2 = 7.44...$
   $x_3 = 7.9097...$
   $x_4 = 7.92$ (3sf)

4. $x_1 = 8/9$
   $x_2 = 0.81085...$
   $x_3 = 0.82965...$

5. $x_1 = 2.7144...$
   $x_2 = 2.67509...$
   $x_3 = 2.6696...$

6. $x_1 = 2.1602...$
   $x_2 = 2.13004...$
   $x_3 = 2.1285...$

1. $x_1 = 4.1231...$
   $x_2 = 5.0490...$
   $x_3 = 5.40333...$

2. $x_1 = 2.6684...$
   $x_2 = 2.78789...$
   $x_3 = 2.793005...$
   $x_4 = 2.79322...$

3. $x_1 = 5$
   $x_2 = 3$
   $x_3 = 4.12310...$

4. $x_1 = 6.3330...$
   $x_2 = 6.22309...$
   $x_3 = 6.2259...$
   $x_4 = 6.22586...$

5. $x_1 = 4/3$
   $x_2 = 0.814814...$
   $x_3 = 1.557...$
   $x_4 = 0.383...$

6. $x_1 = 2.21336...$
   $x_2 = 1.90897...$
   $x_3 = 1.83966...$
   $x_4 = 1.8227299...$