For a curve C,

$$f'(x) = (x-1)(x+3) \text{ and } f(3) = 1$$

1. Find f(x), the equation of C.
2. Use calculus to find the coordinates of the turning points of C.
3. Use calculus to distinguish between the turning points.
4. Sketch the curve C, labelling the crossing of the y-axis and the turning points.
5. Complete the table below

Domain	Range
$x \epsilon \mathbb{R}$	
$x \epsilon \mathbb{R}^+$	
$-3 \leq x \leq 1$	
$x \geq -3$	
$-6 \leq x < 3$	
$x > 1$	

6. Sketch $y = |f(x)|$, labelling the crossing of the $y - axis$ and the turning points.
7. Sketch $y = f|x|$, labelling the crossing of the $y - axis$ and the turning points.
8. Complete the table of turning points below

Curve	Local maximum	Local minimum
$f(x+3)$		
$f(x)+3$		
$f(-x)$		
$-f(x)$		
$2f(x)$		
$f(2x)$		

9. Show that $f(x) = 0$ can be arranged to give the iterative formula

$$x_{n+1} = \sqrt[3]{24 + 9x_n - 3{x_n}^2}$$

10. For x_1=3, calculate the values of x_2, x_3 and x_4 to 3 decimal places.
11. By choosing a suitable interval, show that 2.91 is a root of $f(x) = 0$ correct to 2 decimal places.

Extension

- Write down the sum and the product of the roots of $f(x) = 0$
- Find the other two roots.

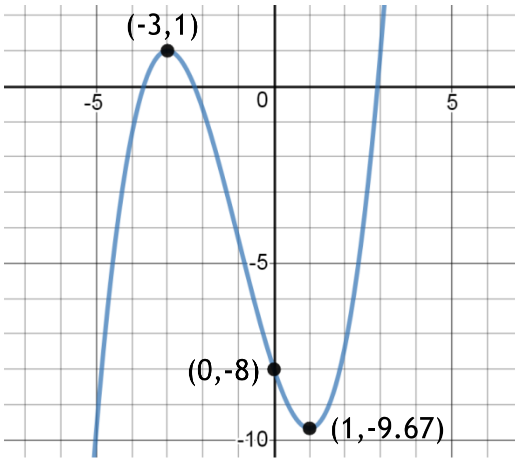
Teacher notes

1. $f(x) = \frac{1}{3}x^3 + x^2 - 3x - 8$

2. (-3, 1) and (1, $\frac{-29}{3}$)

3. Local maximum (-3, 1) and local minimum (1, $\frac{-29}{3}$)

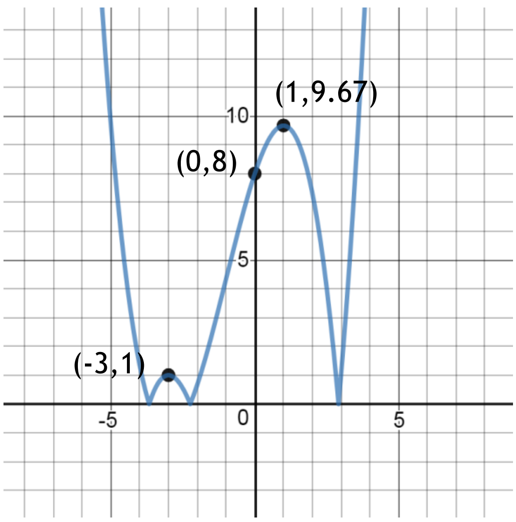
4.



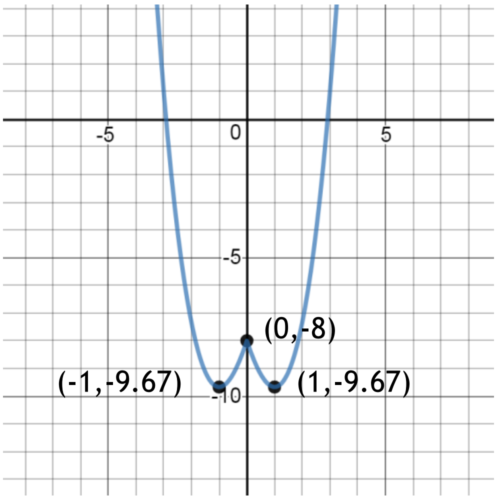
5. Use calculus to distinguish between the turning points.

Domain	Range
$x \epsilon \mathbb{R}$	$-\infty < f(x) < \infty$
$x \epsilon \mathbb{R}^+$	$\frac{-29}{3} \leq f(x) < \infty$
$-3 \leq x \leq 1$	$\frac{-29}{3} \leq f(x) \leq 1$
$x \geq -3$	$\frac{-29}{3} \leq f(x) < \infty$
$-6 \leq x < 3$	$-26 \leq f(x) < 1$
$x > 1$	$f(x) > \frac{-29}{3}$

6.



7.



8.

Curve	Local maximum	Local minimum
$f(x)$	**(-3, 1)**	**(1, $\frac{-29}{3}$)**
$f(x+3)$	**(-6, 1)**	**(-2, $\frac{-29}{3}$)**
$f(x)+3$	**(-3, 4)**	**(1, $\frac{-20}{3}$)**
$f(-x)$	**(3, 1)**	**(-1, $\frac{-29}{3}$)**
$-f(x)$	**(1, $\frac{29}{3}$)**	**(-3, -1)**
$2f(x)$	**(-3, 2)**	**(1, $\frac{-58}{3}$)**
$f(2x)$	**(-$\frac{3}{2}$, 1)**	**($\frac{1}{2}$, $\frac{-29}{3}$)**

9. Rearrangement to

$$x_{n+1} = \sqrt[3]{24 + 9x_n - 3{x_n}^2}$$

10. $x_2 = 2.884$, $x_3 = 2.924$ and $x_4 = 2.911$

11. $f(2.905) < 0$ and $f(2.915) > 0$ and hence 2.91 is a root accurate to 2 decimal places.

Extension - following on from resource 23444 - Investigating quadratic equations…

For the cubic equation $ax^3 + bx^2 + cx + d = 0$

- The sum of the roots is $\frac{-b}{a}$ = -3
- The product of the roots is $\frac{-d}{a}$ = 24
- Students may find these two answers surprising!
- The other 2 roots are -3.67 and -2.24, accurate to 2 decimal places.

www.mathsisfun.com/algebra/polynomials-sums-products-roots.html

The 'cubic formula' - mathworld.wolfram.com/CubicFormula.html

Graphs credit: © 2019 Desmos, Inc.