

Here is a leaflet for a car hire company:




Congruent Car Hire, Heathrow Airport				
“We always have the car to meet your needs”				
Welcome to Congruent Car Hire. Below are our 2019 prices. We look forward to handling your requests, in person, by phone or online!				
Category	Up to 7 days	7 days or more	Compulsory Insurance	Maximum Mileage
A 	£19.95 per day	£18.45 per day	£5.95 per day	80km per day
B 	£21.45 per day	£20.95 per day	£6.95 per day	100km per day
C 	£24.95 per day	£22.45 per day	£7.95 per day	Unlimited
Please note: Excess mileage charged at £0.20 per kilometre, on return of car. A second driver is charged at £4.95 per day.			Special Offer - 5% discount if you book online!!!!	

1. Tariq visits the company office at Heathrow and arranges to hire a category B car for 6 days. He will be the only driver, and he is convinced he will only drive a total of about 500km. How much will the company charge him?
2. Tariq took the offer in question 1, but when he brought the car back, it transpired that he had driven 783km. What excess mileage charge did he have to pay?
3. Mr and Mrs Brown live in Australia and have planned a trip to the UK. They want to have a car available to them as soon as they land at Heathrow, so book a car online. They both want to be able to drive the car. They decide on a category C car, and arrange a three-week hire. How much do they pay?
4. Because the Browns live in Australia, when they pay online they pay in Australian dollars. At the time they paid the exchange rate was £1 = AUD1.79. In addition to this, the credit card they used levied a 2% surcharge for currency transactions. How many Australian dollars did the Browns pay, to the nearest dollar?

Congruent Cars has a membership scheme. You pay £100 per year and this entitles you to a free upgrade by one category (in other words you pay category A prices for a category B car, or category B prices for a category C car). This offer doesn't apply to the compulsory insurance.

5. Mrs Tan, a businesswoman in Hong Kong, makes short but frequent trips to the UK and hires a category C car from Congruent Cars. She is thinking about taking up the membership offer, but wants to figure out if it is something that will save her money in the long run. What calculations would she have to do? What kinds of things would she have to take into consideration? When, exactly, would it become a good idea to take up the membership?

Other car hire firms have offices at the airport. Here is a flyer one of them uses for advertising:

Matrix Motors - everything arranged nicely!			
Category	Cost per day	Compulsory Insurance	Maximum Mileage
<p>A</p> 	£20	£50 for the period of hire	100km per day
<p>B</p> 	£24	£60 for the period of hire	120km per day
<p>C</p> 	£29	£75 for the period of hire	Unlimited

Please note:
Excess mileage charged at £0.25 per kilometre, on return of car.
A second driver is charged at £6 per day.

6. Mr Jumat decides to hire a car with Matrix Motors. He hires a Toyota Tangent (a category C car) for 6 days.
- What is the **total** cost of the hire?
 - What is the average daily cost of the hire?
 - What percentage of the cost is insurance?
7. Mr and Mrs Ronson decide to hire a Ford Function (a category A car). They both want to drive it, and they have calculated that their 10 day trip will involve exactly 900km. Which company - Congruent Cars or Matrix Motors - would be the cheaper option for them, and by how much?
8. T is the total cost for an individual hiring a category C car from Matrix Motors for D days. Write down a formula connecting T and D.
9. Fred is planning a 10-day journey which will involve covering a long distance. Is going to hire a car from Matrix Motors. He originally thought of taking a category B car, but he thinks it might be cheaper to take a category C car because of the unlimited mileage. How many km would Fred have to drive so that the category C car becomes the cheaper option?

Teaching notes

Solutions

- The 500km total is within the allowance, so the total charge is $6 \times (\pounds 21.45 + 6.95) = \pounds 170.40$
- A 6-day hire includes 600km free. This means he has driven 183km over the allowance. This incurs an extra charge of $183 \times \pounds 0.20 = \pounds 36.60$
- A 3-week hire means the daily rate is $\pounds 22.45$, so the cost is $21 \times (22.45 + 7.95 + 4.95) = 742.35$. Booking online means 5% discount. $0.95 \times 742.35 = \pounds 705.23$
- The total AUD cost would be $705.23 \times 1.79 \times 1.02 = 1287.61$ or **AUSD1287** (nearest dollar)
- The offer she would be tempted by would see her pay category B prices for the category C car.
If she usually hired the car for less than 7 days, membership would save her $\pounds 24.95 - \pounds 21.45$ per day.
This is a $\pounds 3.50$ saving per day. $\pounds 100 \div \pounds 3.50 = 28.6$. This means that, if she hires for **29 days or more**, the $\pounds 100$ membership charge results in her saving money.
If she usually hired the car for 7 days or more, membership would save her $\pounds 22.45 - \pounds 20.95$ per day.
This is a $\pounds 1.50$ saving per day. $\pounds 100 \div \pounds 1.50 = 66.7$. This means that, in order for the membership fee to save her money, Mrs Tan would have to drive if she hires for **67 days or more**.
- The total cost is $6 \times \pounds 29 + \pounds 75 = \pounds 249$
 - The average daily cost is therefore $\pounds 249 \div 6$ or **$\pounds 41.50$**
 - The insurance represents $75/249 \times 100\%$ of the total cost. In other words, **30.1%**
- Congruent Cars will charge the following:
Basic Hire = $10 \times \pounds 18.45 = \pounds 184.50$
Insurance = $10 \times \pounds 5.95 = \pounds 59.50$
Mileage excess = 100km, costing $100 \times \pounds 0.20 = \pounds 20$
Second driver = $10 \times \pounds 4.95$
This is a total of **$\pounds 313.50$** or, if they book online, they will pay $0.95 \times \pounds 313.50 = \pounds 297.83$

Matrix Motors will charge the following:
Basic hire = $10 \times \pounds 20 = \pounds 200$
Insurance = $\pounds 50$
Second driver = $10 \times \pounds 6 = \pounds 60$
They would be allowed 1000km free mileage, so a 900km journey incurs no cost.
This is a total of **$\pounds 310$**

Matrix Motors represents a **$\pounds 3.50$** saving compared with Congruent Cars if booked in person but the cheapest option is booking Congruent Cars online for a saving of **$\pounds 12.17$** .
- The formula is $T = 29D + 75$
- The maximum distance you could drive a category B car, without incurring excess mileage charges is 1200km.
If Fred drives K kilometres in 10 days (where $K > 1200$), the category B charge would be:
 $10 \times \pounds 24 + \pounds 60 + \pounds 0.25 \times (K - 1200) = \pounds 300 + \pounds 0.25(K - 1200) = 0.25K$

The category C charge would be $10 \times \pounds 29 + \pounds 75$ or $\pounds 365$

For the category C charge to be more economical, we require: $0.25K > 365$

This means $0.25K > 365$ giving $K > 1460$

So if Fred is going to drive more than **1460km**, it would make more sense to upgrade to category C.

Suggested Extension Work

A. Other tariff options and styles could be put forward and used.

One example:

£23 per day for days 1 to 6,
then £21.35 per day for days 7 to 13,
then £19.95 per day for days 14 onwards

This would be interesting to look at graphically, leading to piece-wise functions.

B. Another option - Saturday and Sunday daily rates are lower than weekday rates.

C. Another feature to include - if the driver is aged between 18 and 25, the insurance rate is higher. There is a nice (statistical) discussion to be had around this.

D. From the outside, at least, car hire prices seem to be very high. It would be a good discussion to have as to what a hire company's overheads might be - salaries, ground rents, business licences, insurance, maintenance, cleaning costs, advertising could all be discussed.