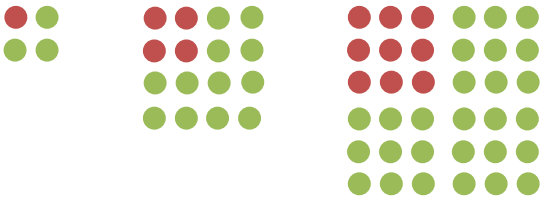
$$\frac{1}{3}, \frac{1+3}{5+7}, \frac{1+3+5}{7+9+11}, \ldots$$

1. Continue this sequence for another 3 terms. What do you notice about the value of each term?

2. How does the diagram below explain the sum of the numbers in the numerators of the sequence?

3. What is the connection between the geometric sequence below and the value of each term of the original sequence?



4. Continue the sequence below for a further three terms and find the pattern of the value of each term

$$\frac{1+3+5}{7+9+11}, \frac{7+9+11}{13+15+17}, \ldots$$

5. What do you notice about this sequence?

$$\frac{(1+3+5)-(7+9+11)}{(13+15+17)-(19+21+23)}, \frac{(3+5+7)-(9+11+13)}{(15+17+19)-(21+23+25)}, \ldots\ldots$$

Teacher notes

1.

$$\frac{1}{3}$$

$$\frac{1+3}{5+7}=\frac{4}{12}=\frac{1}{3}$$

$$\frac{1+3+5}{7+9+11}=\frac{9}{27}=\frac{1}{3}$$

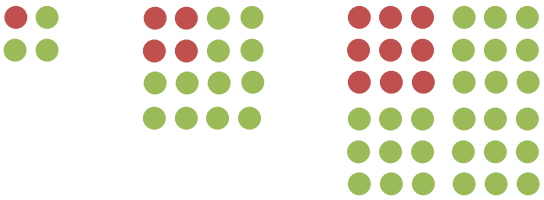
$$\frac{1+3+5+7}{9+11+13+15}=\frac{16}{48}=\frac{1}{3}$$

$$\frac{1+3+5+7+9}{11+13+15+17+19}=\frac{25}{75}=\frac{1}{3}$$

$$\frac{1+3+5+7+9+11}{13+15+17+19+21+23}=\frac{36}{108}=\frac{1}{3}$$

2. Each numerator totals to the square numbers

3. The red dots represent the numerators, the green dots the denominators and the ratio of red to green is always $\frac{1}{3}$



4. The numerators are the odd numbers 1, 3, 5,…the denominators are the odd numbers 3, 5, 7,….

$$\frac{1+3+5}{7+9+11}=\frac{1}{3}\quad\frac{7+9+11}{13+15+17}=\frac{27}{45}=\frac{3}{5}\quad\frac{13+15+17}{19+21+23}==\frac{5}{7}$$

$$\frac{19+21+23}{25+27+29}=\frac{63}{81}=\frac{7}{9}\qquad\frac{25+27+29}{31+33+35}=\frac{81}{99}=\frac{9}{11}$$

5. Each term evaluates to 1

$$\frac{(2n+1+2n+3+2n+5)-(2n+7+2n+9+2n+11)}{(2n+13+2n+15+2n+17)-(2n+19+2n+21+2n+23)}=\frac{-18}{-18}=1$$

Links for further investigation:

- projecteuclid.org/download/pdf_1/euclid.mjms/1316032777
- www.mcm.edu/academic/galileo/ars/arshtml/mathofmotion1.html
- www.amazon.co.uk/Proofs-without-Words-Exercises-Classroom/dp/0883857006