

The Goldbach conjecture - Every **even** whole number **greater than 2** can be written as the sum of two primes.

4 =	6 =	8 =	10 =	12 =
14 =	16 =	18 =	20 =	22 =
24 =	26 =	28 =	30 =	32 =
34 =	36 =	38 =	40 =	42 =
44 =	46 =	48 =	50 =	52 =
54 =	56 =	58 =	60 =	62 =
64 =	66 =	68 =	70 =	72 =
74 =	76 =	78 =	80 =	82 =
84 =	86 =	88 =	90 =	92 =
94 =	96 =	98 =	100 =	

The 'lesser-known' Goldbach conjecture - Every odd positive integer can be written in the form $p + 2a^2$, where p is prime and $a \geq 0$. Example $27 = 19 + 2 \times 2^2$.

Is this conjecture true? If not, identify the odd numbers between 1 and one hundred which cannot be written this way.

1 =	3 =	5 =	7 =	9 =
11 =	13 =	15 =	17 =	19 =
21 =	23 =	25 =	27 =	29 =
31 =	33 =	35 =	37 =	39 =
41 =	43 =	45 =	47 =	49 =
51 =	53 =	55 =	57 =	59 =
61 =	63 =	65 =	67 =	69 =
71 =	73 =	75 =	77 =	79 =
81 =	83 =	85 =	87 =	89 =
91 =	93 =	95 =	97 =	99 =

Using exactly four 4s create sums with solutions from 1 to 100. You may use any of the operators +, - \times and \div , indices, brackets, square roots, factorial notation, $.4 = 0.4444 \dots$ and concatenation, for example $0 = 44 - 44$.

1 =	2 =	3 =	4 =	5 =
6 =	7 =	8 =	9 =	10 =
11 =	12 =	13 =	14 =	15 =
16 =	17 =	18 =	19 =	20 =
21 =	22 =	23 =	24 =	25 =
26 =	27 =	28 =	29 =	30 =
31 =	32 =	33 =	34 =	35 =
36 =	37 =	38 =	39 =	40 =
41 =	42 =	43 =	44 =	45 =
46 =	47 =	48 =	49 =	50 =
51 =	52 =	53 =	54 =	55 =
56 =	57 =	58 =	59 =	60 =
61 =	62 =	63 =	64 =	65 =
66 =	67 =	68 =	69 =	70 =
71 =	72 =	73 =	74 =	75 =
76 =	77 =	78 =	79 =	80 =
81 =	82 =	83 =	84 =	85 =
86 =	87 =	88 =	89 =	90 =
91 =	92 =	93 =	94 =	95 =
96 =	97 =	98 =	99 =	100 =

Teacher notes

The Goldbach conjecture remains one of the most famous unproven problems of mathematics. Between 2000 and 2002, Faber and Faber offered a \$1 000 000 prize to anyone who could prove the conjecture. No one was able to do so and the prize went unclaimed.

You can test any even number using this online tool:

<https://www.dcode.fr/goldbach-conjecture>

The 'lesser-known' Goldbach conjecture was in the last paragraph of a letter Christian Goldbach wrote to Leonhard Euler on 18th November 1752.

eulerarchive.maa.org/correspondence/letters/OO0878.pdf

Several counter examples are known to exist showing the conjecture to be false. Counter examples between 1 and ninety nine are 1, 3 and 17. At that time, the number 1 was considered to be prime, which would allow 1 and 3 to follow the rule.

The French mathematician **Pierre-Simon Laplace** is credited with saying "Read **Euler**, read **Euler**, he is the master of us all" 5777 and 5993 are part of the sequence of 'Stern numbers' - prime numbers which *cannot* be written in the form $p + 2a^2$, where p is prime and $a > 0$. The sequence, which is probably finite, is listed at oeis.org/A060003.

Four 4s

Possible solutions can be found here:

www.pleacher.com/mp/puzzles/mathpuz/mobfour4.html